Name:	Shi Lingsheng
Supervisor:	Prof. Dr. Hans Jürgen Prömel
Field of Research:	Discrete Mathematics
Topic:	Ramsey Theory
PhD Student	at the program since July 2000

Results

A subset S of an abelian topological semigroup G is *syndetic* if there exists a compact set $K \subset G$ so that for any $g \in G$, there exists $k \in K$ with $gk \in S$. The gap of S, $g(S) = \min |K|$. A subset of natural numbers is syndetic if it can be arranged as an increasing sequence $s_1 < s_2 < ...$ with bounded gaps $s_{n+1} - s_n$. Such sets have sometimes been called *relatively dense* sets. A subset of the natural lattice $\mathbb{N}^m$ is *thick* if it contains arbitrarily large sublattices $[\mathbf{a}_n, \mathbf{a}_n + n\mathbf{e}]$, $n \to \infty$. A syndetic set nontrivially intersects each thick set and vice versa. A subset of $\mathbb{N}^m$ is *piecewise syndetic* if it is the intersection of a thick set and a syndetic set. In 1968, Brown [1, 2] first proved that *for any finite coloring of natural numbers there exists a monochromatic piecewise syndetic subset.* In [7], I extended it to higher dimentions. The following are three equivalent versions of our theorem.

For any finite coloring of the natural lattice there exists a monochromatic piecewise syndetic subset.

For all natural numbers k and $f \in \mathbb{N}^{\mathbb{N}}$, there exists (a least) integer $n = n(f, k)$ so that for any k-coloring of $[n\mathbf{e}]$, there is a monochromatic set S with $|S| > fg(S)$. Moreover, $n(f, 1) = \lceil (f(0) + 1)^{1/m} \rceil$ and $n(f, k) \leq (k \max\{f(i) | i \in [n(f, k-1)^m]\})^{1/m} + 1$.

For any finite coloring of a piecewise syndetic set there exists a monochromatic piecewise syndetic subset.

A subset $V \subset \mathbb{N}^m$ is VDW if for any finite $F \subset \mathbb{N}^m$ we have some homothetic copy $\mathbf{a} + bF \subset V$, $\mathbf{a} \in \mathbb{N}^m$, $b \in \mathbb{N}$. Syndetic set is VDW (Proposition 2.8 in [3]) and piecewise syndetic subset of natural numbers is VDW (Theorem 14.1 in [4]). I had a slight stronger result in [7]: *Piecewise syndetic set is VDW.* A syndetic set may be regarded as a discrete analogue of a set without empty interior in a compact Hausdorff space. Then the dual of a thick set corresponds to that of a dense subset. A piecewise syndetic set is the analogue of a set dense in some open set. The third version of our theorem is a discrete analogue of Baire's theorem on unions of nowhere dense sets.

A topological space X is *van der Waerden* [5] if for any sequence (x_n)

in X there exists a convergent subsequence (x_{n_k}) so that (n_k) is VDW. Van der Waerden space is obviously sequentially compact. A topological space X is *Gallai* if for any function $f : \mathbb{N}^m \to X$ there exists a point x in X so that for any neighborhood U of x there exists a VDW subset V of $\mathbb{N}^m$ for which the image under f, $f(V)$ is in U. Gallai space is obviously limit point compact. By the definition, Gallai's theorem may be stated as *finite space is Gallai.* In [6], I extended it to that *if any closed seperable subspace of X is compact then X is Gallai. So is compact space.* Applying to compact metric space, we have a bit stronger result than Theorem 2.9 in [3]. Namely, *if X is a compact metric space then for any function $f : \mathbb{N}^m \to X$ there exists a point x so that for any $\epsilon > 0$ and finite set $F \subset \mathbb{N}^m$, we can find a homothetic copy $bF + \mathbf{a}$ for which the image under f, $f(bF + \mathbf{a})$ is in the ball $B(x, \epsilon)$.* This can be used as a tool in diophantine approximation. It is known [5] that there exists a compact Hausdorff, sequentially compact, separable space with countable basis at all points but one, which is not van der Waerden. So Gallai space need not be van der Waerden. But *if any closed seperable subspace of a Gallai space X is first countable then X is van der Waerden. So is first countable Gallai space.* It is well known that first countable compactness implies sequential compactness. Now, we know inbetween comes van der Waerden space. Namely, *first countable compact space is van der Waerden.*

Activities

- The seventh annual international computing and combinatorics conference in Guilin, China, August 20 to 23, 2001

- Fall School "Discrete Geometry - Triangulations from various points of view" in Alt Ruppin, Germany, October 4 to 6, 2001

- "The Sharpest Cut" Workshop in Honor of Manfred Padberg on the occasion of his 60th Birthday, Konrad-Zuse-Zentrum für Informationstechnik Berlin, Germany, October 11 to 13, 2001

- Block - Courses "Randomized Algorithms" and "Topological Methods in Combinatorics and Geometry" in ETH Zürich, Switzerland, October 22 to November 23, 2001

- 28. Berlin Algorithm Day, Institut für Mathematik der Technischen Universität Berlin, Germany, February 15, 2002
- Seminar "Algorithms and Complexities" at HU (with talks on "Monochromatic finite union of infinite sets" and "Van der Waerden spaces")
- Lectures and colloquium of the European graduate program

References

[1] T. C. Brown, *On locally finite semigroups*, Ukraine Math. J. 20 (1968), 732-738.

[2] T. C. Brown, *An interesting combinatorial method in the theory of locally finite semigroups*, Pacific J. Math. 36 (1971), 285-289.

[3] H. Furstenberg, *Recurrence in Ergodic Theory and Combinatorial Number Theory*, Princeton University Press, Princeton, 1981.

[4] N. Hindman and D. Strauss, *Algebra in the Stone-Čech compactification*, De Gruyter Berlin, 1998.

[5] Menachem Kojman, *Van der Waerden spaces*, Proceedings of The American Mathematical Society, 130, No. 3, (2002), 631-635.

[6] Shi Lingsheng, *Gallai spaces*, script.

[7] Shi Lingsheng, *Monochromatic piecewise syndetic sets*, script.