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Supervisor: Prof. Dr. Rolf H. Möhring
Field of Research: Combinatorial Optimization
Topic: Dynamic Flows in Traffic Networks
PhD Student at the program since October 2000

Field of Research

I am working on optimization problems arising in the area of road traffic. One goal of my work is to find a realistic and at the same time tractable model to describe traffic.

Road traffic can be modeled as flow in a graph representing the road network. Special features of traffic flow are that we have transit times on the arcs, which measure the time it takes to traverse a street, and that we allow flow variation over time. This leads to the notion of *dynamic network flows*, which was introduced by Ford & Fulkerson [**?**].

Given a network $(G = (V, A), u, s, t)$ and travel times τ_a on the arcs, a mapping $f : A \times \{0, \dots, T\} \to \mathbb{R}_0^+$ is said to be a *feasible dynamic flow* if

$$\sum_{a \in \delta^-(v)} f_a(\theta - \tau_a) - \sum_{a \in \delta^+(v)} f_a(\theta) = 0 \qquad \forall v \notin \{s, t\},\ 0 \leq \theta \leq T$$

$$0 \leq f_a(\theta) \leq u_a \qquad \forall a \in A,\ 0 \leq \theta \leq T$$

The interpretation of a dynamic flow f is that $f_a(\tau)$ units of flow are entering the arc a at time τ. Thus, dynamic flows provide the two features mentionened above: we have transit times on the arcs and flow variation over time. Ford & Fulkerson could show in [**?**] that a maximum dynamic flow can be computed in polynomial time.

We are considering a generalization of dynamic flows, namely dynamic flows with *flow-dependent* transit times. In order to model car traffic behavior, transit times should depend on the current flow situation. In our model, we assume that the transit time of an arc is a monotone increasing, convex function of the inflow rate of the arc. In this setting, we can derive approximations for the quickest flow problem. The idea of the proof is to reduce the problem to a quickest dynamic flow problem in an expanded graph with *constant* transit times.

One disadvantage of this model is that *FIFO (First In First Out) - violations* can occur. Traffic flow violates the FIFO-conditions, if flow entering an arc at time θ leaves this arc earlier than flow that had entered this arc before θ. Since this behavior is not acceptable for road traffic, we have to make sure that solutions fulfill the FIFO-conditions. Because of the special structure of our approximative solution, we can guarantee that FIFO-conditions hold.

Further on, we have implemented the data structures to build up time-expanded graphs and to test dynamic flow algorithms.

Activities

- Fall school of the graduate program *Discrete Geometry – Triangulations from various points*, Altruppin, October 4 – 6, 2001
- Talk at the Workshop of the Munich Graduate Program "Angewandte Algorithmische Mathematik" on *Analysis and Optimization* in Munich, Germany, October 8 – 10, 2001
- Blockcourses of the graduate program on *Randomized Algorithms* and on *Topological Methods in Combinatorics and Geometry*, Zürich, October – November 2001
- Talk at the Monday lectures of the graduate program, Berlin, January 21, 2002
- Lectures and colloquia of the graduate program

Preview

- 6-month-stay in Budapest as a guest of the Eötvös University, supervisor András Frank

Literatur

[1] L.R. Ford, D.R. Fulkerson. *Flows in Networks*, Princeton University Press, New Jersey, 1962.