Name:	Carsten Lange
Supervisor:	Prof. Günter M. Ziegler
Field of Research:	Discrete and Combinatorial Geometry
Topic:	Differential Geometry and Combinatorics
PhD student	at the program between January 1, 2000 and December 31, 2001

Field of Research and Results. It is possible to derive upper bounds on the diameter of CW-complexes, which have a strict positive Ricci curvature in the original sense of Robin Forman, [**?**]. Unfortunately, this result cannot be used to estimate the diameter of arbitrary convex polytopes, since not all polytopes have strict postive Ricci curvature; e.g. the dodecahedron and the icosahedron are Ricci-flat. Can this technique be extended to the generalised (weighted) notion of Ricci curvature? The answer to this question is twofold: yes and no. I succeeded in generalising Forman's technique to the general setting, but it was quite disappointing to realise that strict positive Ricci curvature does not suffice in general. Upper bounds can be derived for some examples, where the original method does not work, but there are examples of polytopes with strict positive weighted Ricci curvature, where the method fails (till now). There are still many questions in this direction.

Between October and new year's eve, I spent three month in Emo Welzl's group at ETH Zürich. I had a very warm reception and I learned many different new things during my stay. I started a joint project with Dr. Joachim Giessen, which deals with the reconstruction problem of closed planar curves with no self-intersection from a point sample. There is an associated bounded Voronoi graph to the point sample, whose edges are weighted by a certain distance function. The Laplacian of this weighted graph has nonnegative eigenvalues and the eigenvalue zero has always multiplicity one. An eigenvector of the first positive eigenvalue defines a partition of the vertex set of the Voronoi graph, e.g. consider the nodes corresponding to negative and nonnegative coefficients of this eigenvector. The "nonnegative vertices" induce a subgraph of the original Voronoi graph. Iterating this procedure gives us an ordering of the points which reconstruct the sampled curve. Naturally, the "density" of the sample has to be sufficiently high for the reconstruction to be correct. We hope to give a satisfying explanation for the correctness of this algorithm in near future.

Activities.

- Lectures and Colloquia of the graduate program in Berlin and Zürich

- Participation at the EuroConference on Combinatorics, Graph Theory and Applications, Barcelona, September 12-15, 2001
- Participation at DMV-Jahrestagung, Wien, September 16-22, 2002
- Participation at the graduate program's fall school "Discrete Geometry - Triangulations from various points of view" , Alt-Ruppin, October 3-7, 2001
- Participation at Klausurtagung des Sonderforschungsbereichs 288, St. Marienthal, Februar 25 - March 1, 2002
- Talk at DMV-Jahrestagung, "Kombinatorische Krümmungen und ein Gauß-Bonnet für CW-Komplexe", September 21, 2001
- Talk at Emo Welzl's Mittagseminar, "On diameters of some simple polytopes - an approach by Robin Forman", October 30, 2001
- Talk at Oberseminar Diskrete Geometrie, "Deriving upper bounds on the diameter of some simple polytopes", February 6, 2002
- Talk at SFB-Klausurtagung, "Curvatures and Combinatorics", March 1, 2002
- Course "Pseudorandomness and Algebraic Methods in Combinatorics and Algorithms" by Tibor Szabo, ETH Zürich
- CGC-Blockcourse "Topological Methods in Combinatorics and Geometry" by Jiri Matousek, ETH Zürich
- *Oberseminar Diskrete Geometrie* at TU Berlin
- Kolloquium "100 Jahre Berliner Mathematische Gesellschaft"
- Euler-Vorlesung 2001
- Berliner Algorithmen-Tag (February 15, 2002)

Perspectives. Since January 1, 2002, I am member of Sonderforschungsbereich 288, where I shall continue my research project and finish my PhD.

References

[For00] R. Forman, *Bochner's Method for Cell Complexes and Combinatorial Ricci Curvature*, Preprint, 2000, `http://math.rice.edu/~forman/`

[Lan01] C. Lange, *Combinatorial Bochner-Laplacians, Curvatures and Weitzenböck Formulae*, TU Berlin, September 2001, 16 pages