

Semester Report WS01/02 of Martin Kutz

Name: Martin Kutz
Supervisor: Prof. Dr. Martin Aigner
Field of Research: Algorithmic Graph Theory,
Combinatorial Game Theory
Topic: Games on Hypergraphs
PhD Student: at the program since June 2001

Previous Work and Results

Throughout the first eight months in the program, I have worked on several independent subjects from the field of graph theory and combinatorics.

Treeifying posets Manuel Bodirsky, also PhD student of the program, had transformed a variant of an open question in computer linguistics into a graph theoretic setting: *When can a partial order be extended to a tree, while certain given pairs of elements remain incomparable?* Together we worked on this problem and finally came up with an algorithm that constructs a solution in quadratic time, provided one exists.

Light switch configurations This little sideline project was launched by Dennis Epple, student at the Freie Universität Berlin, as a coffee break math problem: *There is a standard setup for operating a single electric light from several switches. Is it best possible?* We developed a graph theoretic model for such electric light switch circuits and investigated this question in greater depth. To our surprise, we were actually able to prove a sharp lower bound on the number of elementary devices in such circuits and could thus answer that question in the affirmative.

Lucas chains Already before I had come to Berlin, I had written up the main results of my diploma thesis [Kut00] about Lucas chains for journal publication. Lucas chains are a variant of addition chains [Knu98]. They were first considered by Montgomery [Mon] and later rediscovered for public-key cryptography [YL95].

Throughout the review process of my manuscript [Kut], several new aspects of this topic have turned up. I had to engage in Lucas chains again,

in order to understand the algebraic background better and to judge their cryptographic relevance.

Current Research

My current focus of interest is combinatorial game theory as it is being developed in the seminal books [Con76] and [BCG82].

I am investigating the structure of biased positional games on hypergraphs, considered for example in [HJ63]. By now, nothing is known about the complexity of such games. It might well be possible that due to their asymmetry, they are not PSPACE-hard as the unbiased version [Rei80]. In fact, several observations suggest that at least for the class of 3-uniform hypergraphs, the set of winning positions lies in NP.

Activities

- Attended the Fall School “Discrete Geometry - Triangulations from various points of view”, from Thursday to Saturday, October 4 to 6, 2001 in Alt Ruppin (close to Berlin).
- Talk on “Treeifying Posets with Incomparability Constraints” in the colloquium of the graduate program on January 28, 2002.

Preview

I will present the results of my joint work with Manuel Bodirsky at the 19th International Symposium on Theoretical Aspects of Computer Science (STACS), March 14 to 16, 2002 in Antibes Juan-les-Pins.

References

- [BCG82] Elwyn R. Berlekamp, John H. Conway, and Richard K. Guy. *Winning Ways for your mathematical plays*, volume 1,2. Academic Press, 1982.
- [BK02] Manuel Bodirsky and Martin Kutz. Pure dominance constraints. In *Proceedings of the 19th Annual Symposium on Theoretical Aspects of Computer Science (STACS)*, 2002. to appear.

- [Con76] John H. Conway. *On Numbers and Games*. AK Peters, 1976.
- [HJ63] A.W. Hales and R.I. Jewett. Regularity and positional games. *Transactions of the American Mathematical Society*, 106:222–229, 1963.
- [Knu98] Donald E. Knuth. *The Art of Computer Programming*, volume 2, section 4.6.3., pages 461–485. Addison-Wesley, Reading, MA, third edition, 1998.
- [Kut] Martin Kutz. Lower bounds for lucas chains. unpublished.
- [Kut00] Martin Kutz. Grundlegende Betrachtungen zu einer Variante von Additionsketten. Master’s thesis, Rheinische Friedrich-Wilhelms-Universität Bonn, Germany, 2000.
- [Mon] Peter L. Montgomery. Evaluating recurrences of form $X_{m+n} = f(X_m, X_n, X_{m-n})$ via Lucas chains. unpublished, `ftp.cwi.nl:/pub/pmontgom/Lucas.ps.gz`.
- [Rei80] Stephan Reisch. Gobang ist PSPACE-vollständig. *Acta Informatica*, 13:59–66, 1980.
- [YL95] Sung-Ming Yen and Chi-Sung Laih. Fast algorithms for LUC digital signature computation. In *IEE Proceedings—Computers and Digital Techniques*, volume 142 (2), pages 165–169, March 1995.