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Supervisor:	Prof. dr. Martin Aigner
Field of Research:	Graph theory and Combinatorics
Topic	graph polynomials and topological graph theory
Postdoc Student	at the program since November 1 2000

Field of Research and results

This semester we studied a graph polynomial introduced by Arratia, Bollobás, and Sorkin [1]. They call this graph polynomial the interlace polynomial of a graph. It is a generalization of a polynomial defined for 2-in-2-out directed graph, which contains information about the number of ways such a directed graph can be decomposed into k Eulerian cycles.

We found an explicit formula for this polynomial, and could relate it to the Martin polynomial of isotropic systems [2]. We could also solve a conjecture of Arratia, Bollobás, and Sorkin about the evaluation of the interlace polynomial at -1. Furthermore we could give some nice properties of the interlace polynomial. A paper is in preparation.

This semester I also tried to find a new polynomial time algorithm for checking if a graph is linkless or not, but have not succeeded yet; see [4] for the definition of linkless. It comes down to the truth of the following interesting conjecture. Let $\mathcal{C}$ be the regular cell-complex obtained from G by attaching to each circuit a 2-cell, and let $\mathcal{D}$ be the regular cell-complex obtained from $\mathcal{C} \times \mathcal{C} \setminus \Delta$ by identifying (x, y) with (y, x). Here Δ denotes space of all points (x, x) with $x \in \mathcal{C}$. The conjecture says that if G Kuratowski connected, then the second Betti number of $\mathcal{D}$ equals 1 (with coefficient group the integers mod 2). Kuratowski connectivity is a notion introduced in [4]. I will continue with trying to find a proof for this conjecture.

From November 18 to November 23 2001 Rudi Pendavingh, TU Eindhoven, was invited to FU Berlin. We succeeded to show that graphs that can be embedded on the Klein bottle satisfy the following property. First a definition. If G is a simple graph, then $\mathcal{C}_2(G)$ denotes the regular 2-cell complex obtained from G by attaching to each circuit a 2-cell. If G can be embedded on the Klein bottle, then $\mathcal{C}_2(G)$ can be mapped in 4-space in generic position such that each pair of disjoint 2-cells D_1 and D_2 the intersection number of D_1 with D_2 is even. This provides using a theorem of [3] another proof that $\lambda(G) \leq 5$ if G can be embedded on the Klein bottle.

Activities

- I gave a talk at the colloquium of the graduate college on 17 December 2001. .
- referee for SIAM Journal on Discrete Mathematics.
- I gave talks at the Combinatorics seminar at the FU Berlin.
- The following two papers have been accepted for publication: 'Graphs with Magnetic Schrödinger Operators of Low Corank' in Journal of Combinatorial Theory, Series B, and 'On the "largeur d'arborescence"' in Journal of Graph Theory.

Preview

Contact has been made with Oxford (one of the partner universities), which is the place I want stay my last half year.

Literatur

[1] R. Arratia, B. Bollobás, and G.B. Sorkin, *The Interlace Polynomial: A New Graph Polynomial*, Proceedings of the Eleventh Annual ACM-SIAM Symposim on Discrete Algorithm, 237–245, San Francisco, California, 2000.

[2] A. Bouchet, *Tutte-Martin Polynomials and Orienting Vectors of isotropic Systems*, Graphs and Combinatorics **7** (1991), 235–252.

[3] H. van der Holst, and R. Pendavingh, *The Colin de Verdière parameter and 4-dimensional flatness of graphs* , working paper.

[4] N. Robertson, P. Seymour, and R. Thomas, *Sach's Linkless Embedding Conjecture*, Journal of Comb. Theory, Series B, **64** (1995), 185–227.