Name:	Britta Broser
Supervisor:	Dr. Ulrich Kortenkamp, Prof. Helmut Alt
Field of Research:	Complexity Theory and Geometry
Topic	Complex Tracing
PhD Student	at the program since October 1, 2001

Field of Research

During the last months, I have considered the homotopy method for polynomial root-finding from [1]. Here the task is to find an element $z \in \mathbb{C}$ with $|f(z)| < \epsilon$ for a polynomial f and a fixed error bound ϵ. Assuming exact real computations, this method needs $O(d(d + \ln \frac{1}{\epsilon}))$ Newton iterations, where d is the degree of f.
More general homotopy or continuation methods are well known and play an important role in scientific computing. A homotopy is a continuous deformation of one map into another one.
The basic idea of this method is the following: If we want to compute a root of a map f we can choose a suitable map g with known roots and use a homotopy between f and g in order to retract a root of g to a root of f. If all the maps are smooth enough, you can interpret this retraction as a solution of a certain differential equation, which you try to "follow" in discrete time steps.
I could correct some little mistakes and inaccuracies in [1].
The outlook is to use the ideas of this method for some questions arising in the context of the geometry software *Cinderella*. Here, geometric constructions are represented by Geometric Straight-Line Programs (GSP). They consist of free points and dependent elements (like a line connecting two points, the intersection point of two lines, one of the two angular bisectors of two intersecting lines, one of the at most two intersection points of a line and a circle). An instance of a GSP is an assignment of fixed values to all free parameters and choices.
One obvious problem arising from this setup is the *tracing problem*: Given is a movement $\phi(\lambda)$, $\lambda \in [0, 1]$, of the free points on continuous real paths starting at the positions $\phi(0)$ of the free points at an instance $A = A(0)$, such that we can find for every position $\phi(\lambda)$ a matching instance $A(\lambda)$ and $A(\lambda)$, $\lambda \in [0, 1]$ is again continuous. Decide whether a fixed instance B can be the final instance $A(1)$ of the motion. In [4] is shown, that this problem is NP-hard.

Sometimes, for example in automatic theorem proving or for avoiding singularities, it might be useful to consider complex paths. But in contrast to the real situation, the complexity of *complex tracing* is still unknown.
This problem seems to be related to the homotopy method from [1] described above, which has a "good" complexity. My task is to use the ideas from [1] in order to determine the complexity of complex tracing.
For this reason, I am also increasing my knowledge in Projective Geometry ([2], [5]).

Activities

- Lectures and Colloquia of the graduate program
- *Mittagsseminar Theoretische Informatik* at FU-Berlin, including a talk on February 14, 2002, titled *A Homotopy Method for Polynomial Root-Finding*
- Workshop *Computer Algebra in Geometric Computings*, October 1-5, 2001, Lorentz Center in Leiden, The Netherlands
- Lecture *Graphentheoretische Algorithmen* by Günter Rote at FU-Berlin
- Berliner Algorithmen-Tag (February 15, 2002)

References

[1] L. Blum, F. Cucker, M. Shub, S. Smale, *Complexity and Real Computation*, Springer 1998

[2] J. N. Cederberg, *A Course in Modern Geometry*, Springer 1989

[3] U. Kortenkamp, *Foundations of Dynamic Geometry*, PhD-thesis, ETH Zürich, 1999

[4] J. Richter-Gebert, U. Kortenkamp, *Complexity Issues in Dynamic Geometry*, Proceedings of the Smale Fest 2000, 2001

[5] J. Richter-Gebert, *Grundlagen Geometrischer Operationen*, manuscript of a lecture, 2000