Semester Report SS05 of Florian Zickfeld

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Field of Research:	Discrete Mathematics
Topic:	Schnyder Woods and Geometric Representations of Graphs
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Field of Research

Schnyder Woods on Orthogonal Surfaces

Let $V \subset \mathbb{N}^3 \subset \mathbb{R}^3$ be an antichain with respect to the dominance order. The *filter* generated by V is $\langle V \rangle = \{\alpha \in \mathbb{R}^3 \mid \alpha \geq v$ for some $v \in V\}$. The boundary of $\langle V \rangle$ is an *orthogonal surface* S_V. We will call an orthogonal surface degenerate if there are $u, v, w \in V$ and $\{i, j, k\} = \{1, 2, 3\}$ such that

$$u_i = v_i, v_j = w_j, \text{ and } w_k = u_k.$$

For the sake of brevity we will omit to give a definition of *Schnyder Woods* on 3-connected planar graphs here. It can be found e.g. in [2].

It is a well known result by Miller [3], that every non-degenerate orthogonal surface gives rise to a Schnyder Wood S on a weakly 3-connected graph G. We will sketch the construction, following [1]. The vertices will be the elements of V. Let $u, v \in V$, and $u \vee v \in S_V$ and assume for a moment, that there are no elements $x, y \in V$ such that $x \vee y = u \vee v$. Then, the two line segments $[u, u \vee v]$ and $[u \vee v, v]$ will be contained in S_V. At least one of the two segments will be parallel to a coordinate axis. If $[u, u \vee v]$ is parallel to the x_i-axis and $[u \vee v, v]$ is parallel to the x_j-axis this defines a bidirected edge colored i from u to v and j from v to u. If $[u, u \vee v]$ is parallel to the x_i-axis and $[u \vee v, v]$ is not parallel to any coordinate axis, then the edgee uv will be unidirected and colored i.

If we drop the assumption that u, v are the only elements of V that are dominated by $u \vee v$, there is a choice of which edge shall be in the Schnyder Wood. An orthogonal surface without such a choice will be called a rigid orthogonal surface.

Results

All of the following results are joint work with Stefan Felsner. We reproved Theorem 1 ([1]). The proof follows a quite inituitive approach which uses the "split"-operation introduced in [4]. The Brightwell-Trotter Theorem can be concluded from the stated theorem, and we feel that our approach gives the least technical proof for the Brightwell-Trotter Theorem so far.

Theorem 1. *For every Schnyder Wood S on 3-connected planar graph G there is a non-degenerate, rigid orthogonal surface $\mathcal{O}$ such that S is the unique Schnyder Wood associated with $\mathcal{O}$.*

The surface constructed in our proof of Theorem 1 is in general not coplanar, i.e. there is no $c \in \mathbb{R}$ such that the minima lie on the plane defined by $x_1 + x_2 + x_3 = c$. The following example shows that this is not by accident.

Result 1. *There is a Schnyder Wood S_0 on a graph G with ten vertices, such that an orthogonal surface $\mathcal{O}$ associated to S_0 can be either rigid or coplanar, but not both.*

We also give a characterization of non-degenerate, coplanar surfaces in terms of *face weights.*

Result 2. *Let $\mathcal{O}$ be an orthogonal surface which is coplanar and such that the three special vertices of the associated Schyder Wood S have coordinates $(c, 0, 0), (0, c, 0), (0, 0, c)$ for some $c \in \mathbb{N} \setminus \{0\}$. Then, there exist integer face weights w_f, $f \in \mathcal{F}$ for the bounded faces of S such that $\mathcal{O}$ will be the orthongonal surface obtained from the face-count embedding of S.*

Activities

During the last semester, I

- attended the lecture "Diskrete Strukturen"at TU Berlin
- organized and attended the Noon Seminar of the workgroup "Diskrete Mathematik" at TU Berlin
- gave a talk on Theorem 1 at the seminar of the workgroup "Combinatorial Optimization & Graph Algorithms" at TU Berlin

- attended the seminar "Graphen Zeichnen" of the workgroup "Diskrete Mathematik"
- attended the Monday lectures and colloquia of the CGC
- attended the "Einsteintag", a one-day-workshop on recent educational policy in Germany (organized by the Studienstiftung des deutschen Volkes).

Preview

I am planning to attend

- the Summer School on Geometric Combinatorics, Vienna, 18.-29. July '05
- a Summer Academy organized by the Studienstiftung des deutschen Volkes, Rot an der Rot, 6.-20. August '05
- the Eurocomb 05, Berlin, 5.-9. September '05
- the 13th Symposium on Graph Drawing, Limerick, 12.-14. September '05
- CGC-Workshop, Hiddensee, 25.-28. September '05.

References

[1] S. Felsner. Geodesic embeddings of planar graphs. *Order*, 20:135–150, 2003.

[2] S. Felsner. *Geometric Graphs and Arrangements.* Vieweg, 2004.

[3] E. Miller. Planar graphs as minimal resolutions of trivariate monomial ideals. *Documenta Math.*, 7:43–90, 2002.

[4] M. Mosbah N. Bonichon, S. Felsner. Convex drawings of 3-connected plane graphs.