Scientific report of Tzvetalin S. Vassilev

Name:	Tzvetalin S. Vassilev
Supervisor:	Günter Rote
Field of Research:	Computational Geometry
Topic:	Optimal Area Triangulation
PhD Fellow	at the program from May to July 2005

Field of Research

We consider a set S of n points in the two-dimensional Euclidean plane. The points are in *general position*, i.e., no three points are collinear, and no four points are co-circular. In addition we assume that the points do not move, and the size of the set does not change, thus the point set can be considered fixed. Under these premises, a *triangulation* of the point set is a maximal set of non-intersecting straight line segments with vertices in S. For a point set there are an exponential number, $t(S)$, of possible different triangulations. Currently, the best established bounds for $t(S)$ are [1]:

$$\Omega(2.33^n) \leq t(S) \leq O(59^{n-\Theta(\log n)})$$

Therefore, computing a triangulation with a specific properties is not a trivial task from an algorithmic point of view. By *optimal triangulation* we usually mean a triangulation of the point set that achieves an optimum for a given function. For example, if the function we consider is the total length of the edges of the triangulation, the triangulation(s) that achieve(s) the minimum total edge length is known in the literature as Minimum Weight Triangulation (MWT). Computing the Minimum Weight triangulation of a set of points is one of the classical problems in Computational Geometry, whose complexity status is unknown [4]. The function that we optimize is also called *quality measure*. It is reasonable to expect that, depending on the chosen quality measure, the optimal triangulations will be different and thus will have different geometric and structural properties. The algorithmic techniques used to compute particular optimal triangulation may vary as well.
In the special case when the point set S is in *convex position*, i.e. the points of S form a convex polygon, there is a dynamic programming algorithm by Klincsek [9] that computes the optimal triangulation of S with respect to any *decomposable* quality measure in $\Theta(n^3)$ time and $\Theta(n^2)$ space. In the general case there is a general algorithmic technique, called *edge insertion* [2]

that computes the optimal triangulation, in polynomial time, for a class of quality measures (strictly smaller than the class of the decomposable quality measures), that satisfy the so-called *anchor conditions*. There are problem specific polynomial time algorithms that compute some of the well-known optimal triangulations as the *Delaunay* triangulation [10], the *Greedy* triangulation [12], and the MinMax edge length triangulation [11].
In this research we considered the two quality measures based on the areas of the triangles in the triangulation: MaxMin Area triangulation, which maximizes the area of the smallest area triangle over all possible triangulations of the point set, and the MinMax Area triangulations, which minimizes the area of the largest area triangle over all possible triangulations of the point set. The two area-based quality measures are decomposable, thus the Klincsek's result is applicable in the convex case. In the general case, no result has been reported prior to 2001. It was mentioned in one of the survey books [3] that MinMax Area is a hard open problem in the field of optimal triangulations. Our studies revealed geometric, structural and algorithmic properties of these two optimal triangulations, the results are summarized in the following subsection.

Results

- MaxMin and MinMax Area triangulations of a convex set can be computed in $O(n^2 \log n)$ time and $O(n^2)$ space [8]
- The MaxMin Area decision problem can be solved in $O(n^2 \log \log n)$ time and $O(n^2)$ space [5]
- The relative neighbourhood graph of a point set is part of every 30°-triangulation of the point set if such exists [7]
- The optimal area triangulations can be approximated in polynomial time, using angularly-constrained triangulations [6]

Activities

- Preparation of the final version of the Ph.D. Thesis, submitted to the College of Graduate Studies and Research at the University of Saskatchewan on 15 June 2005

- Preparation of the paper "The Relative Neighbourhood Graph is a Part of Every 30°-triangulation" for submission in a refereed journal

- Presentation of the paper "Optimal Area Triangulation with Angular Constraints" at the XI Encuentros de Geometría Computacional (XI Spanish Meeting on Computational Geometry), held in Santander, Cantabria, Spain, 27–29 June, 2005

- Participation in the 21st Annual ACM Symposium on Computational Geometry, held in Pisa, Tuscany, Italy, 6–8 June, 2005

- Talk in the Colloquium of the Graduate Program "Combinatorics, Geometry, Computation": "Optimal Area Triangulations, Part I", Technische Universität, Berlin, 9 May 2005

- Talk in the Noon Seminar of the "Theoretical Computer Science" workgroup: "Optimal Area Triangulations, Part II", Freie Universität, Berlin, 12 May 2005

- Talk in the Noon Seminar of the "Theoretical Computer Science" workgroup: "Shortest Paths in Geometric Graphs", Freie Universität, Berlin, 24 May 2005

- Attendance to the Monday Lectures and Colloquia of the Graduate Program "Combinatorics, Geometry, Computation"

- Attendance to the Noon Seminars of the "Theoretical Computer Science" workgroup at Freie Universität, Berlin

Preview

- Ph.D. Thesis defence, University of Saskatchewan, Saskatoon, Saskatchewan, Canada, 15 August 2005

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