Semester Report SS05 of Taral Guldahl Seierstad

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Supervisor(s):	Prof. Dr. Hans Jürgen Prömel
Topic:	Random graph processes
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Field of Research

My field of research is random graph process. Mostly I have been studying the minimum-degree random graph process, see [1]. Let $G_{\min}(n, 0)$ be the empty graph on n vertices, and for $M > 0$ let $G_{\min}(n, M + 1)$ be obtained from $G_{\min}(n, M)$ by first choosing a vertex of minimum-degree in $G_{\min}(n, M)$ uniformly at random, and then connecting it by an edge to another vertex chosen uniformly at random from the remaining vertices in $G_{\min}(n, M)$. We say that a property holds *asymptotically almost surely* if it holds with probability tending to one as n tends to infinity. We set $t = M/n$, and we consider the graph $G_{\min}(n, tn)$.

The previous semester, Dr. Mihyun Kang and I proved that the minimum degree graph process exhibits a *phase transition* similar to the standard random graph $G_{n,p}$. We found a value $h_g \approx 0.8607$, such that if $t < h_g$, then asymptotically almost surely the largest component in $G_{\min}(n, tn)$ has $O(\log n)$ vertices, whereas if $t > h_g$, then there is a unique giant component with $\Theta(n)$ vertices, while any other component in the graph has $O(\log n)$ vertices.

This semester we have finished writing our article with this result. Furthermore, we have continued to study the minimum-degree process. We have proved that when $t = h_g$, the largest component in $G_{\min}(n, tn)$ has asymptotically almost surely $\Theta(n^{2/3})$ vertices, and we are studying the behaviour of $G_{\min}(n, tn)$ when $t \sim h_g$, which is called the *critical phase*. It appears that in this phase the minimum-degree graph process behaves similar to $G_{n,p}$ does in many ways, and I believe we will be able to prove this.

I have also studied other aspects of the minimum-degree graph process, such as the Hamiltonicity and the cycle distribution of the graph.

Activities

- Attended weekly lectures and colloquia of the CGC. (23 May 2005 I held a talk titled *The giant component in the minimum degree graph process.*)
- Attended weekly seminar of the research group *Algorithmen und Komplexität* at the HU Berlin. (1 April 2004 I held a talk titled *The giant component in the mindegree process.*)
- Attended Learn- & Workshop "Random Graphs and Probabilistic Methods" at HU Berlin, 7 to 9 March 2005. (8 March I held a talked titled *The minimum degree multi-graph process.*)
- Attended the Spring School of the CGC on Enumerative Combinatorics in Netzeband, 1 to 4 June 2005.
- Attended the Berlin-Poznan Seminar in Discrete Mathematics, 4 June 2005. (I held a talk titled *The giant component in the minimum degree graph process.*)
- Attended the IPCO XI Summer School at TU Berlin, 6 to 7 June 2005.

Preview

I will continue to study the phase transition of the min-degree random graph process together with Dr. Mihyun Kang. Our first goal is to describe the structure of $G_{\min}(n, M)$ for $M = h_g n + s$, where $s = o(n)$. Another problem I have been a little involved in, and which I plan to study more next semester is that of counting, exactly or asymptotically, the number of bipartite cubic graphs.

References

[1] Mihyun Kang, Youngmee Koh, Tomasz Łuczak, and Sangwook Ree. The connectivity threshold for the min-degree random graph process. submitted.