

Semester Report SS05 of Maïke Buchin

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Supervisor: Helmut Alt
Field of Research: Computational Geometry
Topic: Fréchet Distance of Triangulated Surfaces
PhD Student in the program since May 2003

Field of Research and Results

The first half of this semester I spent at the ETH Zurich in the group of Emo Welzl. During this time I worked with Joachim Giesen on optimization of triangular meshes. We considered the situation where we are given a fixed point set, e.g. sampled from a surface in \mathbb{R}^3 , and we want to find a triangulation of this point set which best resembles the underlying surface. One possibility for choosing the triangulation is to start with an initial triangulation and to locally optimize a cost function over the triangulation. As cost function we looked at the total absolute discrete Gaussian curvature. Minimizing the total absolute discrete Gaussian curvature yields nice results in the sense that sharp edges and smooth parts of a surface appear in the triangulation as sharp and smooth, respectively, as possible.

Minimizing absolute discrete Gaussian curvature was first applied for optimization of triangular meshes by Lyuba Alboul and Ruud van Damme [1]. They showed that for convex point sets the global minimum of the total absolute Gaussian curvature is always achieved using a simple flip heuristic. For general point sets the flip heuristic does not always reach the global minimum. The question we were interested in was whether an efficient algorithm exists which always finds the global minimum. Our conjecture was that this problem is NP-hard and we showed that for terrains it is [3] by giving a reduction from maximum non-intersecting line segments [2].

Another question I worked on this semester is how to define an average Fréchet distance of curves. The motivation for this is the following: the Fréchet distance of two curves is defined as the maximum pointwise distance minimized over arbitrary reparametrisations. By taking the maximum pointwise distance all other pointwise distances are ignored. If one uses the Fréchet distance for matching, this may not always lead to the desired result. E.g. consider the curves in Figure 1: When matching f to either g or h one might prefer to match f to g rather than h because overall they seem more

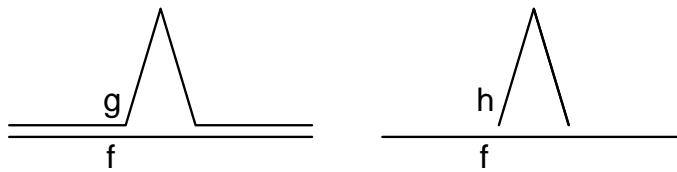


Figure 1: Pairs of curves with equal Fréchet Distance but different overall similarity

similar. But the Fréchet distance between f and g and f and h is equal and after a small perturbation might even be smaller between f and h .

Our goal is therefore to define an average or accumulated Fréchet distance which takes into account all pairwise distances. Some intuitive definitions for this can be derived by taking the curve integral over the path in the Free-Space-diagram or by taking the curve integral over the curves in image space. These definitions however do not fulfill the triangle inequality and are therefore not metrics.

Activities

Talks

- *Semi-Computability of the Fréchet Distance between Surfaces*
European Workshop on Computational Geometry in Eindhoven, the Netherlands, March 9, 2005
- *Semi-Computability of the Fréchet Distance between Surfaces*
Noon Seminar of the Theory of Combinatorial Algorithms Group at the ETH Zurich, April 12, 2005
- *Minimizing the Total Absolute Gaussian Curvature in a Terrain is Hard*
CGC-Colloquium at the TU Berlin, June 13, 2005
- *Average Fréchet Distance: Definitions which are not Metrics*
Noon Seminar of the Theoretical Computer Science Group at the FU Berlin, June 21, 2005
- *Minimizing the Total Absolute Gaussian Curvature in a Terrain is Hard*
Noon Seminar of the Theory of Combinatorial Algorithms Group at the ETH Zurich, July 8, 2005

Attended workshops and schools

- *Spring School on Computational Geometry*
in Eindhoven, the Netherlands, March 7 and 8, 2005
- *21st European Workshop on Computational Geometry*
in Eindhoven, the Netherlands, March 9 to 11, 2005
- *Spring School on Enumerative Combinatorics*
in Netzeband, June 1 to 4, 2005
- *3rd Gremo's Workshop on Open Problems*
in Kappel am Albis, Switzerland, July 4 and 5, 2005

Attended lectures and seminars

- *Noon Seminar* of the Theory of Combinatorial Algorithms Group
at the ETH Zurich, until May 12, 2005
- *Lecture Approximate Methods in Geometry*
by Bernd Gärtner, Joachim Giesen and Emo Welzl at the ETH Zurich,
March 29 to May 12, 2005
- *Lecture Computational Algebraic Geometry*
by Bernd Sturmfels at the ETH Zurich, April 6 to May 12, 2005
- *Monday Lectures and Colloquia*
of CGC in Berlin since May 17, 2005
- *Noon Seminar* of the Theoretical Computer Science Group
at the FU Berlin since May 17, 2005

Preview

In August I will attend the Canadian Conference on Computational Geometry and present the work I did with Joachim Giesen at the ETH Zurich. After the Conference I will visit Carola Wenk at the University of Texas at San Antonio for two weeks.

In November I will be participating in the GI-Dagstuhl Research Seminar 'Algorithms for Sensor and Ad Hoc Networks'.

References

- [1] L. Alboul and R. van Damme. Polyhedral metrics in surface reconstruction: Tight triangulations. In T. Goodman and R. Martin, editors, *The Mathematics of Surfaces VII*, pages 309–336. Clarendon Press, Oxford, 1997.
- [2] J. Kratochvil and J. Nešetřil. Independent set and clique problems in intersection-defined classes of graphs. *Comment. Math. Univ. Carolinae*, 31:85–93, 1990.
- [3] Maïke Buchin and Joachim Giesen. Minimizing the total absolute gaussian curvature in a terrain is hard. In *17th Canadian Conference on Computational Geometry*, 2005. To appear.