Semester Report SS05 of Oliver Klein

Name:	Oliver Klein
Supervisor:	Prof. Dr. Günter Rote
Field of Research:	Computational Geometry and Combinatorics
Topic:	Matching Shapes with a Reference Point
PhD Student	in the program since April 2003

Field of Research

The topic of my research can be summarized as the analysis of reference points and their usage for shape matching under classes of transformations, like translations, rigid motions (translations and rotations) and similarity operations (rigid motions and scalings). Let $\mathcal{K} \subset \mathbb{R}^d$ be a set of shapes and δ be a distance measure on $\mathcal{K}$. Then a δ-reference point for $\mathcal{K}$ is defined as a Lipschitz-continuous mapping $r : \mathcal{K} \to \mathbb{R}^d$ that is equivariant under the considered class of transformations $\mathcal{T}$. The Lipschitz-continuity of r can be expressed by

$$\forall A, B \in \mathcal{K} \ : \ ||r(A) - r(B)|| \leq c \cdot \delta(A, B).$$

In this context, the Lipschitz-constant c is called the quality of the reference point.

The first problem arising is to find reference points for different sets of shapes $\mathcal{K}$ and distance measures δ. The second challenge is to find ways of using these reference points to construct approximation algorithms based on them. In particular this means to find algorithms which find a transformation Φ^*, such that

$$\delta(A, \Phi^*(B)) \leq \alpha \cdot \min_{\Phi \in \mathcal{T}} \delta(A, \Phi(B)), \tag{1}$$

where $\alpha \in \mathbb{R}^+$ is some constant independent from the concrete choice of A and $B \in \mathcal{K}$. In this context α is called the approximation factor of the algorithm.

The research is mainly based on a paper by Alt, Aichholzer and Rote ([1]). In this paper the case of $\mathcal{K} = \mathcal{C}^2$, the set of compact convex subsets of $\mathbb{R}^2$, and δ as the Hausdorff-Distance δ_H is considered. This distance measure is defined as the smallest ε such that the Euclidean distance from every point of A to its nearest point of B is at most ε and vice versa. Exact algorithms to determine the optimal transformation minimizing the Hausdorff-Distance under translations, rigid motions and similarity operations are known, but unfortunately the running time of these algorithms is not satisfactory for most applications. The authors of [1] develop approximation algorithms using reference points for matching under translations, rigid motions and similarity operations with approximation factors $c + 1$, $c + 1$ and $c + 3$, respectively. Furthermore, it is shown that the Steiner point (Steiner Curvature Centroid) is a δ_H-reference point for $\mathcal{C}^2$ of quality $\frac{4}{\pi}$. It is also shown, that this quality is optimal, which means that there cannot exist any δ_H-reference point with a smaller Lipschitz-constant. This is shown using strong functional-analytic tools and the axiom of choice. Therefore the proof is not constructive.

Summarizing the lower bound on the quality of a reference point of $\frac{4}{\pi}$ and the upper bound of the algorithm using reference points of quality $c+1$ with respect to translations, it seems reasonable that there are shapes $A_1, A_2, ...$ which cannot be matched in a way that

$$\forall i \neq j \ : \ \delta_H(A_i + t_i, A_j + t_j) \leq (1 + \frac{4}{\pi} - \beta) \cdot \delta_H^{opt}(A_i, A_j), \tag{2}$$

where $\delta_H^{opt}(A, B)$ is the optimal Hausdorff-Distance under translations, $\beta \in \mathbb{R}^+$ is any constant and $t_i \in \mathbb{R}^2$ are translation vectors. Observe that under these assumptions the vectors t_i can be interpreted as the reference points of the given shapes. Unfortunately, those shapes A_i, or even their existence, are not known so far. More knowledge about the nature of these shapes would lead to a better understanding of the approximation algorithms and, eventually, to even better algorithms.

Our most recent, more theoretical approach on this problem is based on the non-convex shapes given in [1] showing a lower bound for the quality of the Steiner Point. We could use them to create a small set of three shapes proving a non-convex lower bound for the approximation factor of the translation algorithm of 1.5.

Alt, Behrends and Blömer ([2]) follow a slightly different approach. They define reference points as an equivariant mapping $r : \mathcal{K} \rightarrow \mathbb{R}^d$ which leads, by making the two reference points coincide, to a constant factor approximation (1). We will call such points pseudo-reference points, since this definition is weaker than the one given in [1].

In October 2004, in the early beginning of my time in Utrecht, Remco Veltkamp and I had the idea of applying the reference point approach to weighted point sets with respect to the Earth Mover's Distance (EMD). The Earth Mover's Distance on weighted point sets is a useful distance measure for e.g. shape matching and colour-based image retrieval, see [4], [5] and [6] for more information. The EMD between two weighted point sets A and B can be interpreted as follows: Imagine the weighted points of A as piles of earth and those of B as holes. Basically, the EMD is the physical work needed to move the piles of earth of A into the holes of B, scaled by the minimum total weight of the two sets. Let now EMD_p denote the EMD, when we are regarding the EMD defined on the ground set $\mathbb{R}^d$ with distance measure L_p, $1 \leq p \leq \infty$. Until now we got a couple of interesting results on this. First, we have shown that there is an EMD_p-reference point for weighted point sets with equal total weight, namely the centers of mass of these sets. The quality of this reference point is 1. Second, we have shown that there is also no reference point for weighted point sets with unequal total weights and, even worse, that there is no pseudo-reference point for those sets. We have further shown, that the center of mass is an optimal EMD_p-reference point in the sence that there is no reference with a smaller Lipschitz-constant. Furthermore we have developed approximation algorithms for translations, rigid motions and similarity operations with approximation factors $(c+1)$, $2^{d-1}\sqrt{d}^{d-1}(c+1)$ and $2^d\sqrt{d}^{d-1}(c+1)$, respectively. The running time of these algorithms are $O(T^{EMD_p}(n,m))$ for translations and $O(n^{d-1}m^{d-1}T^{EMD_p})$ for rigid motions and similarity operations, where n and m are the number of points of A and B, and $T^{EMD_p}(n,m)$ denotes the time needed to compute the EMD_p between those two sets. In the important case of the EMD defined on the Euclidean distance in the plane, the approximation factors reduce to 2, $2(c+1)$ and $4(c+1)$ and can be calculated in $O(T^{EMD_2}(n,m))$ for translations and $O(nmT^{EMD_2}(n,m))$ for rigid motions and similarity operations. An upper bound on $T^{EMD_p}(n,m)$ is $O(n^2m^2\log(\max\{n,m\}))$ using a strongly polynomial minimum cost flow algorithm by Orlin ([8]). In practice, of course, using the simplex algorithm to solve the flow problem or a $(1+\varepsilon)$-approximation given by Cabello et al. ([3]) will be much faster.

Quite recently, Cabello et al. ([3]) have been working on similar problems. The advantage of our approach is that the results given can be applied to arbitrary dimension and distance measure on the ground set, even to more than the here mentioned L_p-distances. Therefore the results are widely applicable.

We have also worked on the Proportional Transportation Distance (PTD) and have shown that the center of mass in this case is a PTD-reference point for weighted point sets with arbitrary total weights. Additionally, all the results proven for the EMD can easily be adapted for the PTD.

The results have been sent to ISAAC 2005 in Sanya, Hainan, China, and a technical report has been published, see [7].

The EMD, as defined above for weighted point sets, is an instance of a more general distance function defined on arbitrary subsets of $\mathbb{R}^d$, the so-called Monge-Kantorovich-Distance, see [9] for more details. We have generalized the reference point method to work on subsets of $\mathbb{R}^d$ with respect to this distance measure.

Future Work

There are still a number of open problems. I would like to finish the work started in Utrecht: First of all, the running time and approximation factor of the algorithms for rigid motions and similarity operations do not seem to be optimal. I have some promising ideas on how to improve on this.

Like mentioned above, we have extended the reference point methods to the Monge-Kantorovich-Distance. Unfortunately, like in the case of the EMD, we have to compute the distance between two fixed sets at least once. It seems not clear in the literature, how or in which cases we actually can compute the exact solution or can at least get an approximation on this.

In case of the Hausdorff-distance, the non-convex lower bound on the quality of the Steiner point could be used to get a non-convex lower bound on the approximation factor of the translation algorithm. Lately, we found a new convex lower bound on the quality of the Steiner point. We would like to use these shapes proving this bound to get a lower bound on the approximation factor of the approximation algorithms.

While reading a report by Gerald Weber [10] my attention fell on a slightly stronger definition of regular reference points and some open problems connected to it. In this definition of regular reference points the set of allowed transformations to create the approximation is reduced. Therefore a general reference point need not be regular or it is regular but with a worse approximation factor. However, up to now there is no general reference point known, that is not a regular reference point with the same approximation factor. It may be interesting to find such a reference point.

Activities

Talks

- *Approximation Algorithms for the Earth Mover's Distance Under Transformations Using Reference Points*
  2nd Dutch Computational Geometry Day at Universiteit Utrecht, January 18, 2005

- *Approximation Algorithms for the Earth Mover's Distance Under Transformations Using Reference Points*
  Colloquium of the Center for Geometry, Imaging and Virtual Environments (GIVE) at Universiteit Utrecht, February 24, 2005

- *Approximation Algorithms for the Earth Mover's Distance Under Transformations Using Reference Points*
  21st European Workshop on Computational Geometry at TU Eindhoven, March 9-11, 2005

- *Approximation Algorithms for the Earth Mover's Distance Under Transformations Using Reference Points*
  Noon Seminar of the TI-AG at FU Berlin on April 7, 2005

- *Approximation Algorithms for the Earth Mover's Distance Under Transformations Using Reference Points*
  CGC-Colloquium at HU Berlin on June 27, 2005

- *Minimizing the Earth Mover's Distance Under Rigid Motions*
  Noon Seminar of the TI-AG at FU Berlin on June 30, 2005

Attended events

- *Monday Lectures and Colloquia* of CGC in Berlin

- *Noon Seminar* of the TI-AG at FU Berlin

- *2nd Dutch Computational Geometry Day* at Universiteit Utrecht, January 18, 2005

- *Spring School on Computational Geometry* at Technische Universiteit Eindhoven, March 7-8, 2005

- *21st European Workshop on Computational Geometry* at Technische Universiteit Eindhoven, March 9-11, 2005

- *Spring School on Enumerative Combinatorics* in Netzeband, June 1-4, 2005

- *Berliner Algorithmen Tag* at FU Berlin, July 15, 2005

Long Term Exchange

- Research Stay at Universiteit Utrecht, Institute of Information and Computing Science, Center for GIVE, Workgroup of Prof. Dr. Mark Overmars, Supervisor Dr. Remco Veltkamp, October, 2004 - March, 2005.

Preview

- Attend the 9th Workshop on Future Research in Combinatorial Optimization (FRICO) in Wien, September 2-4, 2005
- Attend the European Conference on Combinatorics, Graph Theory and Applications (EuroComb) at TU Berlin, September 5-9, 2005
- Attend the 5th Workshop on Combinatorics, Geometry and Computation on Hiddensee, September 25-28, 2005
- Attend the Block Course on Combinatorial Optimization at Work at ZIB, October 4-15, 2005

References

[1] H. Alt, O. Aichholzer, G. Rote: 'Matching Shapes with a Reference Point', in Int. J. of Comp. Geom. and Appl., Volume 7, pages 349-363, August 1997

[2] H. Alt, B. Behrends, J. Blömer: 'Approximate matching of polygonal shapes', Proc. 7th Ann. Symp. on Comp. Geom., 1991, 186-193

[3] S. Cabello, P. Giannopoulos, C. Knauer, G. Rote. Matching Point Sets with respect to the Earth Mover's Distance. In *Proceedings of the European Symposium on Algorithms*, 2005.

[4] S. Cohen. Finding Color and Shape Patterns in Images. PhD thesis, Stanford University, Department of Compute Science, 1999.

[5] P. Giannopoulos, R. Veltkamp. A pseudo-metric for weighted point sets. In *Proc. 7th European Conf. Comp. Vision*, LNCS 2352, pages 715–731, 2002.

[6] K. Graumann, T. Darell. Fast contour matching using approximate Earth Mover's Distance. In *Proc. of the IEEE Conf. Comp. Vision and Pattern Recognition*, LNCS 2352, pages I: 220–227, 2004.

[7] O. Klein, R. C. Veltkamp. Approximation Algorithms for the Earth Mover's Distance Under Transformations Using Reference Points. Technical Report UU-CS-2005-003, http://ftp.cs.uu.nl/pub/RUU/CS/techreps/CS-2005/2005-003.pdf, 2005.

[8] J. B. Orlin. A Faster Strongly Polynomial Minimum Cost Flow Algorithm. In *Operations Research*, vol.41,no.2, pages 338–350, 1993.

[9] S. Rachev. The Monge-Kantorovich mass transference problem and its stochastic applications. In *Th. of Prob. and Appl.*, 29, pages 647–676, 1985.

[10] G. Weber: 'The Centroid is a Reference Point for the Symmetric Difference in d Dimensions', Technical Report UoA-SE-2004-1, The University of Auckland, 2004