Semester Report SS05, Cornelia Dangelmayr

Name:	Cornelia Dangelmayr
Supervisor:	Martin Aigner and Stefan Felsner
Field of Research:	Graph Theory
Topic:	Intersection Graphs and Graph Classes
PhD Student	associated member at the program since May 2004

Field of Research

Well known classes of intersection graphs are of families of Jordan curves $\Omega(JC)$ and as special case line segments $\Omega(LS_k)$ in the plane, where k denotes the number of directions the line segments are parallel to. For several classes of planar graphs, constructions are known such that the graphs can be represented as intersection graphs of the given objects. For example, bipartite planar graphs can always be represented as grid intersection graphs $\Omega(LS_2)$, showed by Hartmann et alt. in [2, 5], and triangle-free planar graphs as $\Omega(LS_3)$, showed by Castro et alt. in [5]. We found also constructions for outerplanar and series parallel graphs as $\Omega(LS_3)$. The general question if planar graphs are intersection graphs of line segments $\Omega(LS_k)$ parallel to k directions for any k remains an open problem, and so does the question if all 3-colorable planar graphs are $\Omega(LS_3)$.

For intersection graphs of Jordan curves it is well known, that the recognition problem is NP-hard [10], but it is open whether this problem is decidable. If the objects are line segments there are trivial recognition algorithms [4] but even in this case there is not much known about the structure of the intersection graphs. A rather new result on this is from Fraysseix and Mendez [8]. They give conditions for certain plane graphs to be represented as intersection graphs of Jordan curves and investigate when these representations can be „strechted "to representations of line segments [7]. The condition for Jordan curves is related to a partition of a planar embedded graph with given forbidden subgraphs into subgraphs which can be locally represented as intersection graphs preserving the order of the embedding, such that they can use the embedding of the plane graph to give a global representation of the wanted case. It would be interesting if this methode could be modified in regard to the forbidden subgraphs or for triangulations.

Other interesting fields open when we look at the structure of intersection graphs itself as geometric objects. Matousek [11] and Solymosi [16] show that there is just a small number of graphs that are interesection graphs of lines resp. line segments in the plane. This result is obtained by counting in how many different ways a set of labelled line segments in the plane can intersect. Furthermore Solymosi [16] describes necessary conditions in terms of subgraphs for graphs that are intersection graphs of line segments and obtains results of Ramsey type for those graphs. One example is that any system of line segments $\mathcal{S}$ in the plane with a minimum number of crossings cn^2 has two disjoint subsystems $\mathcal{S}_1, \mathcal{S}_2 \subset \mathcal{S}$ of size ate least $\frac{(2c)^A}{660}n$, A const., $A \leq 10^6$ and every segment of $\mathcal{S}_1$ crosses every segment of $\mathcal{S}_2$. Other questions may be directed to the given arrangement of line segments and its number of cells for example in terms of extremal questions.

It seems promising to look at graph operations and their relation to intersection representations. Given a graph $G\backslash e$ and its intersection representation $IG(G\backslash e)$, under which conditions can a vertex v_e be split into two neighboured ones u, v where the adjacencies $N(v), N(u)$ are distributed such that the resulting graph is still an intersection graph of line segments or Jordan curves. Observations of this kind may help to understand more of the struture of the intersection representation and relations to given graphs. So we can ask which subgraphs or neighbourhood partitions of $N(v_e)$ do prevent a conversion of $IGG\backslash e$ into $IG(G)$ for the new graph G. So I will further concentrate on this and similar problems and try to find more relations between the objects itself, the representations of the intersection graphs and the graphs itself.

Activities

- I attended the Monday Lectures and Colloquia of the CGC.
- I participated in the spring school in Netzeband, June 1st to 4th 2005.
- I took part in a „Blockseminar "of Prof. Felsner in Bad Freienwalde, July 8th to 10th 2005.
- I took part in the weekly seminar of the group of Prof. Felsner at the TU.

Preview

- summer school in Vienna, July 18th to 29th 2005.

Literatur

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