Semester Report SS04 of Maike Walther

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Supervisor:	Helmut Alt
Field of Research:	Computational Geometry
Topic:	Fréchet Distance of Triangulated Surfaces
PhD Student	in the program since May 2003

Field of Research

I continued my work on the Fréchet distance of triangulated surfaces [2]. Given two parametrised triangulated surfaces $f, g : [0,1]^2 \to \mathbb{R}^3$, I am interested in the computability of their Fréchet distance

$$\delta_F(f,g) := \inf_{\substack{\sigma:[0,1]^2\to[0,1]^2 \\ \text{orient.pres.homeom.}}} \sup_{t\in[0,1]^2} ||f(t) - g(\sigma(t))||$$

which is known to be NP-hard [3].

From a computational point of view, the class of homeomorphisms on the unit square is difficult to handle. One idea how to cope with this, is by a discrete approximation of the Fréchet distance, where the infimum is instead taken over a countable class of functions which have discrete descriptions.

A lower bound of the Fréchet distance can be obtained in such a way using the *Simplicial Approximation Theorem* (e. g. [1]), which says that continuous functions can be approximated by simplicial maps, after perhaps subdividing the domain space.

For an upper bound, the class of simplicial maps needs to be restricted to those that can be approximated by homeomorphisms, i. e. are "close" to homeomorphisms. We propose to restrict to simplicial maps that map the boundary onto the boundary and are bijective on interiors of highest-dimensional simplices, i. e. triangles.

In total, we define the discrete Fréchet distance as

$$\delta_{dF}(f,g) := \inf_{\substack{n,m, \\ s:K^m\to L^n \\ \text{simplicial*}}} \sup_{\substack{\text{vertices} \\ v\in K^m}} ||f(v) - g(s(v))||$$

where K^m and L^n denote the m^{th}- and n^{th}-barycentric subdivision of the underlying triangulations of the unit square for f and g, respectively, and *simplicial** denotes the restricted simplicial maps. For this, we obtain the equality $\delta_F = \delta_{dF}$. In particular, this will yield the Semi-Decidability of the Fréchet distance of triangulated surfaces.

In the last semester I attended four block courses in Prague and Berlin. During the course on permutation groups I worked together with Peter Cameron and others on the irreducibility of the cycle index of a transitive permutation group. Kevin Buchin and I checked the cycle indices of groups up to degree 30 using the GAP and MuPAD software and found 6 transitive groups with reducible cycle index. Based on these examples, Jan Hubicka, Peter Cameron and others proved two propositions, which in particular yield infinitely many transitive groups with reducible cycle index.

Activities

Talks

- *Factorising the cycle index of a transitive permutation group*
  Noon Seminar of the TI-AG at the FU Berlin on April 1

- *Approximating the Fréchet Distance by a Discrete Fréchet Distance*
  Noon Seminar of the TI-AG at the FU Berlin on June 8

- *Discrete Approximation of the Fréchet distance*
  CGC-Colloquium at the TU Berlin on July 5

Block Courses

- *Permutation Groups, Structures, and Polynomials*
  by Peter Cameron at the Charles University in Prague,
  January 26 to March 2

- *Arrangements in Computational and Combinatorial Geometry*
  by Micha Sharir at the Charles University in Prague,
  January 29 to February 27

- *Random Generation and Approximate Counting*
  by Volker Kaibel at the TU-Berlin,
  April 15 to May 21

- *Discrete Geometry: Polytopes and More*
  by Günther Ziegler at the TU-Berlin,
  April 20 to May 19

Attended events

- *Monday Lectures and Colloquia* of CGC in Berlin
- *Noon Seminar* of the TI-AG at the FU Berlin
- Lecture *Discrete Geometry: Polytopes and More* by Günther Ziegler, at the TU Berlin, continuation of the block course, June 15 to 30
- *Berliner Algorithmen Tag* at the TU Berlin on July 12

References

[1] G. E. Bredon. *Topology and Geometry*, volume 139 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, Heidelberg, Berlin, 2 edition, 1995.

[2] Maurice Fréchet. Sur la distance de deux surfaces. *Ann. Soc. Polonaise Math.*, 3:4–19, 1924.

[3] Michael Godau. *On the complexity of measuring the similarity between geometric objects in higher dimensions.* PhD thesis, Freie Universität Berlin, Germany, 1998.