Semester Report SS04 of Taral Guldahl Seierstad

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Supervisor:	Prof. Dr. Hans Jürgen Prömel
Topic:	Random graphs and random greedy processes
PhD Student	at the program since January 2004

Field of Research

This was my first full semester in the program, and I have been studying random graphs and, in particular, random graph processes. The theory of random graphs is presented in [Bol01] and [JŁR00].

We say that a property holds *asymptotically almost surely* if the probability that it holds tends to one, as n tends to infinity.

We consider the following graph process: Let H be a fixed non-empty graph. Every edge in the complete graph K_n on n vertices is assigned a "birthtime" chosen uniformly at random from the interval $[0, 1]$. We then start with the empty graph on n vertices at time zero, and let the time increase gradually. Whenever the birthtime of an edge is reached, the edge is added to the graph if, and only if, it does not form a subgraph isomorphic to H with any previously added edges. We let $M_{n,p}(H)$ be the graph at time p, and $M_n(H) = M_{n,1}(H)$. $M_n(H)$ will be a maximal H-free graph. This process was first studied by Erdős, Suen and Winkler in [ESW95], where they found asymptotically almost sure bounds for the number of edges in $M_n(H)$, when H is a triangle. Osthus and Taraz found similar bounds in [OT01], in the case where H is a strictly 2-balanced graph. (A graph G is strictly 2-balanced if it has at least three vertices and edges, and for every proper subgraph G' of G, we have $\frac{e(G)-1}{v(G)-2} > \frac{e(G')-1}{v(G)'-2}$.)

We would like to find out more about the structure of $M_n(H)$. I have considered the process where H is a triangle, and in particular the problem of which subgraphs $M_n(K_3)$ asymptotically almost surely does or does not have.

I have proved that $M_n(K_3)$ asymptotically almost surely contains a $K_{2,3}$, but does not contain a $K_{t,t}$ for $t = 20000 \log n$, and furthermore that the expected number of copies of $K_{3,3}$ in $M_n(K_3)$ is $\Omega(n^{\frac{3}{2}})$.

It would be nice to reduce the t above to a constant – that is, to find an integer t such that $M_n(K_3)$ asymptotically almost surely does not contain a

$K_{t,t}$. Furthermore, I would like to prove that $M_n(K_3)$ asymptotically almost surely contains a $K_{3,3}$. According to computer experiments it appears likely that $M_n(K_3)$ asymptotically almost surely contains $K_{t,t}$ for $t \leq 4$, but not for $t \geq 5$.

Activities

- Block course "Random generation and approximate counting" by Volker Kaibel at the TU in Berlin.
- Spring School on Combinatorics 2004 in Vysoka Lipa in the Czech Republic from May 3 to May 13. (I held a talk titled *The graph reconstruction problem.*)
- Weekly lectures and colloquia of the CGC.
- Weekly seminar of the research group *Algorithmen und Komplexität* at the HU Berlin. (30 April I held a talk titled *Pfaffian Orientations and Perfect Matchings.*)

Preview

Will attend the 4th Workshop on Combinatorics, Geometry, and Computation, 2004

References

[Bol01] Béla Bollobás. *Random graphs*, volume 73 of *Cambridge Studies in Advanced Mathematics.* Cambridge University Press, Cambridge, second edition, 2001.

[ESW95] Paul Erdős, Stephen Suen, and Peter Winkler. On the size of a random maximal graph. *Random Struct. Algorithms*, 6(2-3):309–318, 1995.

[JŁR00] Svante Janson, Tomasz Łuczak, and Andrzej Rucinski. *Random graphs.* Wiley-Interscience Series in Discrete Mathematics and Optimization. Wiley-Interscience, New York, 2000.

[OT01] Deryk Osthus and Anusch Taraz. Random maximal H-free graphs. *Random Struct. Algorithms*, 18(1):61–82, 2001.