Semester Report SS04 of Dirk Schlatter

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Field of Research:	Random Discrete Structures
Topic:	Planar Graphs
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Field of Research

For the first few weeks of the past semester, I have continued work on the enumeration problem for planar graphs, in which I had become interested around the beginning of the year. The best lower and upper bounds for the set $\mathcal{P}_n$ of all (labelled) planar graphs used to be

$$n!(26.18)^{n+o(n)} \leq |\mathcal{P}_n| \leq n!(32.16)^{n+o(n)},$$

see [1] and [2], respectively. The aim was to improve the upper bound by a double counting argument using spanning trees, but I stopped when Giménez and Noy [3] announced a tremendous improvement on both bounds, using generating function techniques:

$$n!(27.22685)^{n+o(n)} \leq |\mathcal{P}_n| \leq n!(27.22688)^{n+o(n)}.$$

Thereafter, my focus shifted back towards random planar graphs, and more precisely, *triangulation processes*: Starting from an empty graph on n vertices, in each step, choose a random edge (of K_n) and insert it into the present graph if it remains planar. The probability of an edge being inserted at a certain stage in this process is of course highly dependent on the previous choices, quite contrary to the situation in the standard random graph model. It therefore comes as no surprise, that the usual probabilistic methods are difficult to apply to this case.

The main conjecture is that, although the resulting random triangulation is not uniform, it has similar properties (see [4]), e.g. it contains a.a.s. all planar subgraphs of constant size. In order to prove this, a lemma of the following flavour might be quite useful: *After ϵn^2 edges have been considered, the number of edges which can be added in the next step is $o(n^2)$.* I am currently trying to prove such a lemma by considering various planar graph parameters and the effect the insertion of an edge has on them.

Activities

Conferences and Workshops

- April 2 – May 21 CGC Block Course *Random Generation and Approximate Counting* at TU Berlin

Lectures and Seminars

- Weekly lectures and colloquia of CGC
- Weekly seminar of the research group *Algorithmen* at HU Berlin
- Weekly seminar on *Färbung von Graphen* at HU Berlin

Preview

I will continue to work on the random planar graph process described above. Apart from the CGC Workshop in Stels (October 4-7), I plan to visit the DMV Symposium on Discrete Mathematics in Zürich (October 7-8), and then stay with the research group of Angelika Steger at ETH for 4-6 months.

References

[1] A. Bender, Z. Gao and N.C. Wormald, *The number of labeled 2-connected planar graphs*, The Electronic Journal of Cmbinatorics, 9(1):R43, 2002.

[2] N. Bonichon, C. Gavoille and N. Hanusse, *An information–theoretic upper bound of planar graphs using triangulation*, in 20^{th} Annual Symposium on Theoretical Aspects of Computer Science (STACS), volume 2607:499-510 of Lecture Notes in Computer Science, 2003. Springer-Verlag.

[3] O. Giménez and M. Noy, *Estimating the growth constant of labelled planar graphs*, accepted for the Third Colloquium on Mathematics and Computer Science: Algorithms, Trees, Combinatorics and Probabilities, Vienna, September 2004.

[4] C. McDiarmid, A. Steger and D.J.A. Welsh, *Random planar graphs*, Journal of Combinatorial Theory, Series B, accepted for publication.