Semester Report WS03/04 and SS04

Name:	Heiko Schilling
Supervisor:	Prof. Dr. Rolf H. Möhring
Field of Research:	Graph Algorithms
Topic	Algorithms on large and complex Networks
PhD Student	associated member since July 2001

Field of Research and Results

Accelerated Shortest Path Algorithms. In the past two semesters we have been working on fast algorithms for shortest path problems which are at the core of more complex algorithms for route guidance problems in traffic networks and whose running time is therefore of utmost importance (95% of the overall computation time, [JMSS02]). We model these route guidance problems as network flows where one has to solve a multi-commodity min cost flow problem. Formulated as a Linear Program where the variables represent paths in the network, the corresponding separation problem can be interpreted as a (length-bounded) shortest path problem: Find a shortest path with respect to the reduced costs on each arc.

We investigated acceleration methods for (constrained) shortest path computations, like hierarchical separator and the so-called bit-label approaches in the common shortest path case and goal-directed and bi-directed approaches in the constrained case.

For shortest paths, divide and conquer seems to yield already a significant speed-up (factor of up to 42). Here the underlying graph is divided along small separators into regions. A preprocessing phase then gathers information to facilitate the search for shortest paths that stretch over several regions.

Higher accelerations are achieved by the bit-label approach, based on a given embedding of the graph. For every arc a leaving a node v and every region R the information is recorded whether a lies on a shortest path from v to a vertex in region R. This approach combined with a simple rectangular geographic partition of the graph leads to the most significant speed-ups by a factor 200-350 on the German traffic network. This can even be improved if one considers better graph partitions. For this we used *Metis*, a library for multilevel graph partitioning algorithms, and achieved an extra speed-up factor of up to 2. So overall on our traffic networks we can accelerate a standard Dijkstra algorithm by a factor of up to 700.

We also looked at possible combinations of acceleration techniques and the best we accomplished on our traffic networks is to combine bit-labels with the standard bidirectional approach. For long distance requests the thereby accelerated Dijkstra algorithm traverses only slightly more arcs than the number of arcs on the shortest paths found for these requests.

For the constrained shortest path problem standard acceleration methods can also be applied to some extend. Here the simple goal-directed search shows the best speed-up, as the bi-directed variant suffers from the lack of a good stopping criteria.

The previous results are joint work with Ekkehard Köhler and Prof. Möhring and will be submitted to ALENEX 2005.

Together with Thomas Willhalm and Birk Schütz (Uni Karlsruhe), we are looking for other favorable graph partitions (e.g. using Kd-trees) for our bit-label approach as well as for other combinations of acceleration techniques, for instance with the so-called geometric containers, [WW03].

Recently we started a cooperation with Prof. Nagel (ETH Zürich / TU Berlin) and Michael Balmer (ETH Zürich) on our accelerated shortest path algorithms. The goal here is to apply our techniques to a dynamic Dijkstra algorithm they use in their traffic simulation Matsim, [Mat] and [Tra].

Length-bounded $s-t$ –cuts. During my guest visit in Zürich we started working on length-bounded cuts. We call a subset C_E or C_V of the edge set E or node set V of a graph $G = (V, E)$ a *length-bounded $s-t$ –edge–cut* or *–node–cut* with respect to some length bound L if the nodes s and t have distance greater than L in $G' = (V, E \setminus C_E)$ or $G' = (V \setminus C_V, E')$. Here edges or inner nodes reps. of $s-t$ –paths are being counted.

The complexity of finding a minimum length-bounded $s-t$ –cut has been an open question for a long time but we were able to make considerable progress here.

By a gap-preserving reduction to the Minimum Vertex Cover Problem it could be shown that finding a minimum length-bounded $s-t$ –cut is APX-hard for a length bound L of 4 or greater in the edge as well as in the node case. By a different reduction we have shown APX-hardness for a length bound less or equal $|E| - k_E$ or $|V| - k_V$ resp., for some constants $k_E, k_V \in \mathbb{N}_{>1}$.

The only known results so far were that the problem is polynomial solvable for length bounds less or equal 3 in both the node case, [LNLP78], as well as the edge case, [MM03].

Furthermore we are searching for a better approximation algorithm for this problem since the only known result here is a $L + 1 - dist(s, t)$ – approximation, [Bai03].

This work is a collaboration with Georg Baier (TU Berlin), Alex Hall (ETH Zürich), and Thomas Erlebach (ETH Zürich). Our results will be published next year.

Rate-dependent dynamic flow model. Together with Alex Hall we are working on a novel model for "Flows over Time" which captures the behavior of cars traveling through a traffic network better than previous models. Here the transit time of a unit of flow through an arc a is dependent on the flow rate at any point on that arc a. Therefore we call this model "Rate-dependent Flow Model". We showed that it is NP-hard to compute an optimal solution in the new model and developed an LP-based algorithm which we evaluated with several experiments on real world data of traffic networks. Among other things we compared the quality of the solutions with solutions generated by an FPTAS for a related but considerably less realistic model, the so-called "Inflow-dependent Model", [HLS03], as well as the "Load-dependent Model", [KS02]. Our results here will be presented at the Operations Research Conference, 2004, and will be published soon.

Network matching. If given two networks together with a layout, how can one decide whether they are topologically equivalent or not? The motivation for this problem has practical background. Frequently, we are given different input data of traffic networks which actually represents the same geographic area, but it might be given in different resolutions. So that makes it then hard to decide the equivalence question. In case the given networks are topologically equivalent, how can one then match intersections or streets with streets/paths? One goal thereby is to collect all additional data which comes along with several input network data for the same geographic area. We came up with some simple heuristic algorithms but we are still looking for a better solution to this problem. To our knowledge there is one algorithm for these kind of problems, [Pen02], but there is room for improvement. This work is being carried out as part of a collaboration with Michael Balmer.

Activities

Talks

- *Monday Colloquia of the CGC Graduate Program.*
- "Accelerated Shortest Path Algorithms", *Monday Colloquia of the CGC Graduate Program*, Berlin, November 03, 2003.
- "Accelerated Shortest Path Algorithms", *Netzwerktag at DFG Research Center MATHEON*, Berlin, December 15, 2003.
- "Accelerated Shortest Path Algorithms", *DFG Workshop "Shortest Paths"*, Karlsruhe, March 16, 2004.
- "Accelerated Shortest Path Algorithms", *SOS Seminar Prof. Widmayer group and Prof. Erlebach group*, Zürich, May 26, 2004.
- "Accelerated Shortest Path Algorithms", *Mittagsseminar Prof. Wagner group*, Karlsruhe, June 15, 2004.
- "Efficient Algorithms for Path-Based and Dynamic Flow Problems in Large Networks", *Jahreskolloquium of the DFG project "Algorithms on Large and Complex Networks"*, Karlsruhe, July 21–23, 2004.
- "Accelerated Shortest Path Algorithms", *Operations Research*, Tilburg, September 1–3, 2004.

Guest visits

- Guest visit at the groups of Prof. Erlebach and Prof. Nagel, ETH Zürich from May until October.
  (supported by CGC, DFG project 1126 and ETH Zürich)
- Guest visit at the group of Prof. Wagner, Uni Karlsruhe
  from June 14–18 and July 23/24.
- Guest visit at the group of Martin Skutella, MPI Saarbrücken
  on June 18/19.

Miscellaneous

- DFG DVWG Summer school, September 6–10.
- Supervising two tutors at the DFG project "Efficient Algorithms for Path-Based and Dynamic Flow Problems in Large Networks".
- Biweekly meetings of the traffic research group, Prof. Möhring. (now weekly meetings of the COGA group, Prof. Möhring)
- Support of implementation work of several students working on their diploma thesis in the COGA group, Prof. Möhring.

References

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[HLS03] A. Hall, K. Langkau, and M. Skutella. An fptas for quickest multi-commodity flows with inflow-dependent transit times. In *Proceedings of the 6th International Workshop on Approximation Algorithms for Combinatorial Optimization Problems*, 2003.

[JMSS02] O. Jahn, R. H. Möhring, A. S. Schulz, and N. E. Stier Moses. System-optimal routing of traffic flows with user constraints in networks with congestion. Technical Report 754, Technische Universität Berlin, Fachbereich Mathematik, Berlin, Germany, 2002.

[KS02] E. Köhler and M. Skutella. Flows over time with load-dependent transit times. In *Proceedings of the 13th Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 174–183, San Francisco, CA, 2002.

[LNLP78] L. Lovász, V. Neumann-Lara, and M. Plummer. Mengerian theorems for paths of bounded length. *Periodica Mathematica Hungarica*, 9:269–276, 1978.

[Mat] Matsim. http://www.matsim.org.

[MM03] A. R. Mahjoub and S. T. McCormick. The complexity of max flow and min cut with bounded length paths. *unpublished manuscript*, November 2003.

[Pen02] Ram M. Pendyala. Development of GIS-Based Conflation Tool for Data Integration and Matching. 2002.

[Tra] Transims. http://www.acl.lanl.gov.

[WW03] D. Wagner and T. Willhalm. Geometric speed-up techniques for finding shortest paths in large sparse graphs. In Giuseppe Di Battista and Uri Zwick, editors, *Proc. 11th European Symposium on Algorithms (ESA 2003)*, volume 2832 of *LNCS*, pages 776–787. Springer, 2003.