Semester Report SS04 of Anja Krech

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Field of Research:	Discrete Mathematics
Topic:	Problems in the hypercube
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Field of Research

Hamiltonian cycles in the hypercube

The graph of the n-dimensional hypercube can be regarded as the Hasse-diagram of the Boolean lattice $\mathcal{B}_n$ of the subsets of a set of n elements. A well-known open problem is the question whether the graph of the two middle levels of $\mathcal{B}_{2k+1}$ is Hamiltonian for all k. I have been studying the approaches of several authors to solve this problem, in particular the construction of Savage and Winkler [4] of cycles containing roughly 84% of all vertices, the best result so far. Another approach I am investigating is to characterize those n-symbol sequences that can arise as the code of such a Hamiltonian cycle. There already exists a characterisation by Evdokomov and Perezhogin ([2]), and I would like to investigate whether one can use this for the construction of a Hamiltonian cycle.

Turán-type problems in the hypercube

Turán-type problems investigate the maximum number of edges in an H-free subgraph of a graph G. In the classical problem, G is the clique K_n, and the order of magnitude of the maximum is sought for various fixed "forbidden" subgraphs H. I have been studying the corresponding question when, instead of K_n, the base graph is the graph of the n-dimensional cube Q_n, and the special case where the forbidden subgraph is a smaller subcube, i.e., a d-cube for a fixed d. Let $f(n,d)$ be the minimum number of edges one has to delete from Q_n such that no Q_d- subgraph remains and let $c_d = \lim_{n\to\infty} \frac{f(n,d)}{e(Q_n)}$, where $e(Q_n) = n2^{n-1}$, the total number of edges of the n-cube. It is known from [3] that $c_d = \Omega(\frac{\log d}{d2^d})$. In previous work with Tibor Szabó we gave a construction yielding an upper bound for c_d of $\frac{4}{(d+1)^2}$ for odd d, and $\frac{4}{d(d+2)}$ for even d. I have been trying to find new approaches to improve these bounds and want to continue with this work. In the case $d = 3$, I conjecture that $c_3 = \frac{1}{4}$.

Maybe one can show in this context, that if the edges of Q_n are colored with four colors in such a way that every Q_3-subgraph contains an edge of each color, then every color-class must contain roughly $\frac{1}{4}$ of all edges. This would be a generalisation of a result of Bialostocki ([1]) about 2-colorings of Q_n without a monochromatic C_4-subgraph.

Activities

I attended the Monday lectures of the CGC in Berlin and the seminar "Geometrie und Kombinatorik" at the FU Berlin.

References

[1] A. Bialostocki. Some ramsey type results regarding the graph of the n-cube. *Ars Combinat.*, 16-A:39–48, 1983.

[2] A.A. Evdokimov and A.L. Perezhogin. Minimal enumerations of subsets of a finite set and the middle level problem. *Discrete Applied Mathematics*, 114:109–114, 2001.

[3] M. Livingston N. Graham, F. Harary and Q.F. Stout. Subcube fault-tolerance in hypercubes. *Information and Computation*, 102:280–314, 1993.

[4] C. D. Savage and Peter Winkler. Monotone Gray Codes and the Middle Levels Problem. *J. Combin. Theory , Series A*, 70:230–248, 1995.