Semester Report SS04 of Cornelia Dangelmayr

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Field of Research:	Graph Theory
Topic:	Intersection Graphs and Graph Classes
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Background

Let $F = \{S_1, .., S_n\}$ be a family of sets with $S_i \subseteq S$ for $1 \leq i \leq n$. The intersection graph of F, denoted $\Omega(F)$, is the graph having F as vertex set with S_i adjacent to S_j if and only if $i \neq j$ and $S_i \cap S_j \neq \emptyset$. A graph $G = (V, E)$ is an intersection graph of a set S if there exists a family F such that $G \cong \Omega(F)$. In my master thesis I worked on intersection graphs of geometric objects in the plane, especially on families of disks, pseudodisks and rectangles in view of colorability. One approach for this problem is the χ-binding function, that provides with an upper and lower bound for the chromatic number subject to the clique number ω. Taking into account the geometric representation I found χ-binding functions for intersection graphs of families of pseudodisks and assertained, that one of the common methods to obtain χ-binding functions for intersection graphs of families of rectangles was NP-complete. In the course of these investigations a lot of interesting questions about relations between graph classes and the elements of S emerged similiar to the characterisation of planar graphs by coin graphs.

Field of Research

In this sense, my main issue are possible characterisations of intersection graphs of convex objects in the plane by classes of graphs and vice versa.

To get an overview I spent the first weeks acquainting myself with literature and papers of intersection graph theory and graph classes like [2, 6]. One early attempt in this direction is the PhD thesis of E.R. Scheinerman [8], later generally elaborated in [7]. A well known class of intersection graphs is of families of line segments in the plane with $\leq t$ representatives for each vertex placed in k directions $\Omega(LS[k, t])$ or on l parallel lines $\Omega(LS(l, t))$. Among several characterisations by matrices or forbidden subgraphs there do already exist characterisations by classes of graphs for some special cases. Insomuch

do interval graphs form a subfamily of perfect graphs, while bipartite planar graphs can always be represented by grid intersection graphs [1, 3]. At this point one upcoming question is, if this was true for all bipartite graphs or which were necessary respectively sufficient conditions for a representation. With this objective I'm trying to formulate adjacency relations that prevent such grid representations.
If we keep at planar graphs, also for triangle-free planar and outerplanar graphs representations as $\Omega(LS[k,1])$ are known [3, 9, 5]. For the whole class it is known, that a representation as 3-interval graph is possible. To get an idea of the geometric conditions I'm working on a construction of a $LS[k,1]$ representation for outerplanar graphs that may yield methods to work on open questions like:

- How many directions k are necessary for a representation of outerplanar graphs?
- Are all planar graphs presentable as $\Omega(LS[k,1])$ and if so, how many directions k are necessary for the representation?
- Are all 3-colorable planar graphs presentable by $\Omega(LS[k,1])$ with $k \leq 3$ directions?

Concentrating on those and related problems I will also incorporate familiar classes like chordal graphs in view of representations as intersection graphs of line segments and other convex objects in the plane.

Activities

I attend the monday lectures of the CGC and the weekly seminar of Martin Aigner.

References

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