Semester Report SS04 of Kevin Buchin

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Supervisor:	Günter Rote
Field of Research:	Computational Geometry
Topic:	Space-Filling Curves, Probabilistic Analysis of Geometric Algorithms
PhD Student	in the program since May 2003

Field of Research

In this semester, I continued my research on the use of space-filling curves for the probabilistic analysis of geometric algorithms and optimization problems. Furthermore, I attended the block courses of the DOCCOURSE in Prague and Berlin.

In my work I am considering space-filling curves like the Hilbert Curve [3, 4] which have a geometric construction scheme and map the unit interval onto on the d-dimensional unit cube. For points in the unit cube, such a curve defines an ordering of the points by finding a pre-image for every point and linearly ordering the pre-images. The Lipschitz-continuity of order $1/d$ of such a curve yields an $O(n^{1/d})$ bound on the length of the tour through the points along this order for a fixed dimension d.

I used this bound to develop an incremental algorithm for constructing the Delaunay triangulation of points in the unit square. Devroye et al. [2] proved that in a Delaunay triangulation of n uniformly distributed, independent points in the unit square, a fixed line of length l at distance $3\sqrt{\log n/n}$ from the boundary intersects $O(l\sqrt{n})$ triangles in expectation. Therefore, a tour through k points along a space-filling curve order intersects $O(\sqrt{kn}+k)$ triangles in expectation.

When inserting points in rounds as proposed by Amenta et al. [1], this can be used to locate the points of a round in the triangulation constructed so far. I proved that the expected time needed for point location within a round is linear in the size of the round, even if the insertion order within the round is not random. In total, this yields an expected linear time algorithm.

The algorithm treats points near the boundary separately. To overcome this, I am currently examining the properties of random Delaunay triangulations near their boundary. I also continued working on the probabilistic analysis of Euclidean optimization problems and gave a talk in the noon seminar on possiblities of proving central limit theorems for Euclidean functionals.

During the block course in Prague, I worked with Peter Cameron and others on the cycle index of transitive permutation groups. Together with Maike Walther I found several transitive permutation groups with reducible cycle index. This in turn led to two classes of transitive permutation groups, for which Jan Hubicka, Peter Cameron and others could prove that they have reducible cycle index.

Activities

Talks

- *Central Limit Theorems for Euclidean Functionals*
  Noon Seminar of the TI-AG at the FU Berlin on April 6

- *Walking in Delaunay Triangulations*
  Noon Seminar of the TI-AG at the FU Berlin on June 24

- *Insertion Orders for Incremental Construction of Delaunay Triangulations*
  CGC-Colloquium at the FU Berlin on June 28

Block Courses

- *Permutation Groups, Structures, and Polynomials*
  by Peter Cameron at the Charles University in Prague,
  January 26 to March 2

- *Arrangements in Computational and Combinatorial Geometry*
  by Micha Sharir at the Charles University in Prague,
  January 29 to February 27

- *Random Generation and Approximate Counting*
  by Volker Kaibel at the TU-Berlin,
  April 15 to May 21

- *Discrete Geometry: Polytopes and More*
  by Günther Ziegler at the TU-Berlin,
  April 20 to May 19

Attended events

- *Monday Lectures and Colloquia* of CGC in Berlin
- *Noon Seminar* of the TI-AG at the FU Berlin
- Lecture *Discrete Geometry: Polytopes and More* by Günther Ziegler at the TU Berlin, a continuation of the block course, June 15 to 30
- *Berliner Algorithmen Tag* at the TU Berlin on July 12

References

[1] N. Amenta, S. Choi, and G. Rote. Incremental constructions con BRIO. In *Proceedings of the Nineteenth Annual Aymposium on Computational Geometry*, pages 211–219. ACM Press, 2003.

[2] L. Devroye, E. P. Mücke, and B. Zhu. A Note on Point Location in Delaunay Triangulations of Random Points. *Algorithmica*, 22:277–482, 1998.

[3] D. Hilbert. Ueber die stetige Abbildung einer Linie auf ein Flächenstück. *Math. Ann.*, 38:459–460, 1891.

[4] H. Sagan. *Space-Filling Curves*. Springer Verlag, 1994.