Scientific report of Paweł Żyliński

Name:	Paweł Żyliński
Supervisor(s):	Helmut Alt, Günter Rote
Field of Research:	Combinatorial Geometry & Graph Thory
Topic:	Cooperative guards in Art Gallery Problem
PhD Fellow:	at the program from April 2003 to July 2003

Field of Research

During these four months I continued working on the Cooperative Guard Problem in Fortresses and Grids. The context is the following.

The original *art gallery problem* raised by Klee asks how many guards are sufficient to watch every point of the interior of an n-vertex simple polygon. The guard is a stationary point that can *see* any point which can be connected to it with a line segment within the polygon. In 1975 Chvatál [2] proved that $\lfloor \frac{n}{3} \rfloor$ guards are occasionally necessary and always sufficient to cover a polygon of n vertices. Since then many different variations of this problem have arisen; see [7], [8] for more details.

Guards in Fortress One of a family of guard problems, independently posed by Joseph Malkelvitch and Derick Wood, is the *fortress problem*; i.e. one wants to determine the minimal number of guards sufficient to see every point of the exterior of an n-vertex simple polygon (the guard is a stationary point that can see any point which can be connected to it with a line segment without the polygon.) In 1983 O'Rourke and Wood [7] solved the fortress problem for vertex guards – they showed that $\lceil \frac{n}{2} \rceil$ vertex guards are sometimes necessary and always sufficient. A tight bound of $\lceil \frac{n}{3} \rceil$ point guards was given by O'Rourke and Aggarwal [7].

Guards in Grid A *grid* P is a connected union of vertical and horizontal line segments (introduced by Ntafos [6]); a grid can be thought of as an orthogonal polygon with holes, consisting of very thin corridors. A point $x \in P$ can see point $y \in P$ if the line segment $\overline{xy} \subseteq P$. Ntafos established that a minimum cover for a 2D-grid of n segments has $n - m$ guards, where m is the size of the maximum matching in the intersection graph of the grid, and it may be found in $O(n^{2.5})$ time [6]. However, in the case of 3D-grids, the problem of finding the minimum guard set is NP-complete [6].

Cooperative Guard Problem The concept of *cooperative guards* was proposed by Liaw et al. [5]. For a guard set $\mathcal{S}$ we define the *visibility graph* $VG(\mathcal{S})$ as follows: the vertex set is $\mathcal{S}$ and two vertices v_1, v_2 are incident if they see each other. The guard set $\mathcal{S}$ is said to be *cooperative* if the graph $VG(\mathcal{S})$ is connected. The idea behind this concept is that if something goes wrong with one guard, all the others can be informed.

In 1994 Hernández-Peñalver [4] proved that $\lfloor \frac{n}{2} \rfloor - 1$ connected point (vertex) guards are sometimes necessary and always sufficient to cover any point of the interior of an n-vertex polygon.

During this stay we studied the Cooperative Guard Problem for fortresses and grids.

Results

■ Continuing the exploration started at Gdańsk University, we obtained the following theorem:

Theorem 1. Let P be an non-convex fortress of k pockets $p_1, \ldots, p_k$, each of respectively n_{p_i} vertices. Then, $1 + \sum_{i=1}^{k} \lfloor \frac{n_{p_i}-1}{2} \rfloor$ cooperative point guards always suffice to cover all points of the plane exterior to P or of the boundary of P.

■ We showed (joint work with Till Nierhoff, Humboldt University, Berlin) that the minimum cooperative guard problem in a grid (MCGP) can be solved in a polynomial time for both 2-dimensional and 3-dimensional grids. In the first case, MCPG corresponds to the problem of finding a minimum spanning tree in the intersection graph of a grid, thus, a $O(n + k)$ time algorithm is obtained, where n is the number of segments and k is the number of intersections in the grid. In the latter case, an algorithm uses $O(n^{10})$ time; the solution is obtained from a spanning set of a 2-polymatroid constructed from the intersection graph of the grid.

Activities

- Attended the lecture "Algorithmische Geometrie" by Dr. Christian Knauer at FU Berlin.
- Attended the "Monday Lectures and Colloquia" of the Graduate Program.
- Attended the "Mittagsseminar Theoretische Informatik" at FU Berlin.
- Attended the workshop "General ECG Workshop and Workshop on Software", June 25 to 26, 2003 at FU Berlin.
- Attended the "31. Berliner Algorithmen Tag", Juli 11, 2003 at HU Berlin.

Talks:

- "k-guarded guards in art galleries", Mittagsseminar TI, April 22, 2003
- "Cooperative guards in grids", Mittagsseminar TI, July 17, 2003
- "Vertex covers and cooperative guard sets", Colloquium of the Graduate Program, June 16, 2003

Preview

- In my home institute in Gdańsk I will write up all the current results and ideas. They should lead to a nice paper in a forth coming conference.
- In the cooperation together with Till Nierhoff (Humboldt University) and Adrian Dumitrescu (University of Wisconsin-Milwaukee) we intend to find a polynomial algorithm for the Minimum Weighted Cooperative Guard Problem in Grids.
- Attend "Fall School on Computational Geometry", October 2 to 4, 2003 in Neustrelitz.

References

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[8] J. Urrutia, *Art Gallery and Illumination Problems.* in: Handbook on Computational Geometry, Elsevier Science, Amsterdam (2000).

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