# Semester Report SS03 of Maike Walther

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| Name: | Maike Walther |
| Supervisor: | Helmut Alt |
| Field of Research: | Computational Geometry |
| Topic: | The Fréchet Distance as a Similarity Measure for Discrete Geometric Objects |
| PhD Student | at the program since May 2003 |

## Field of Research

During my first semester at CGC I have been looking at similarity measures for discrete geometric objects such as polygonal curves and surfaces. The similarity of such objects is typically measured by defining a metric on them. For parametrized curves and surfaces a natural metric - more natural than the widely used Hausdorff metric - is the Fréchet metric. The Fréchet metric has been studied by Alt and Godau in [AG95] and [God98] and can be defined as follows (cf. [God98]):

**Definitions** We are considering continuous mappings $f : A \to X$ where $(X, \delta_X)$ is a metric space and $A \subset \mathbb{R}^d$ is homeomorphic to $[0,1]^d$ for a fixed dimension $d \in \mathbb{N}$. The Fréchet Distance of two such mappings $f : A \to X$, $g : B \to X$ is defined as

$$\delta_F(f,g) := \underset{\sigma:A\tilde{\to}B}{inf} \; \underset{x\in A}{sup} \; \delta_F\big(f(x),\, g(\sigma(x))\big)$$

where $\sigma : A\tilde{\to}B$ denotes that $\sigma : A \to B$ is an orientation preserving homeomorphism. This is a pseudo-metric and a metric on the equivalence classes $f_{/\sim} = \{g \,|\, \delta_F(f,\, g) = 0\}$. The geometric objects we are interested in are mappings $f : A \to X$ which are *simplicial*, that is $A$ is the underlying space of a simplicial complex and $f$ is affine on each simplex of the complex. Note that for $d = 1$ these mappings are polygonal curves and for $d = 2$ triangle meshes.

H. Alt and M. Godau [AG95] showed that the Fréchet distance of polygonal curves can be computed in polynomial time:

**Theorem 1** *Given two polygonal curves $P, Q$ of size $n, m$ and $\epsilon > 0$, it can be decided in $\mathcal{O}(nm)$ time, whether the Fréchet distance $\delta_F(P,Q) \le \epsilon$. $\delta_F(P,Q)$ can be computed in $\mathcal{O}(nm\,log(nm))$ time.*

In his PhD thesis [God98] M. Godau proved that for higher dimensions the decision problem for the Fréchet metric is NP-hard:

**Theorem 2** *Given two simplicial objects* $f, g$ *of dimension* $d \geq 2$ *and an* $\epsilon > 0$*, deciding whether the Fréchet distance* $\delta_F(f, g) \leq \epsilon$ *is NP-hard. Also the* $\delta$*-approximation of this problem is NP-hard for* $\delta = 0.0000000016$*.*

Theorem 2 gives rise to the following questions:

- Is the decision problem for the Fréchet metric for dimension $d \geq 2$ computable, furthermore is it in NP, i.e. NP-complete?
- Is there a good heuristic or approximation algorithm for computing the Fréchet metric for higher dimensions?

These are the questions I am interested in, whereby I am concentrating on dimension 2 and $X = \mathbb{R}^3$, i.e. the objects I am considering are triangle meshes in $\mathbb{R}^3$. Concentrating on dimension 2 not only seems to be the most natural generalization, it is also an important case for applications. For instance in Computer Graphics the Fréchet distance of triangle meshes would be a suitable error measure for simplifications of triangle meshes.

I have also studied the related question of simplification of polygonal curves under the Hausdorff and Fréchet metric as described in [AG99] and [AHPMW02].

## Activities

I attended the following lectures, colloquia, seminar and workshop:

- Lectures and colloquia of CGC in Berlin
- *Mittagsseminar* of the TI-AG at the FU Berlin with a talk on "Near-linear Time Approximation Algorithms for Curve Simplification in Two and Three Dimensions" on 10.7.03
- Lecture on Computational Geometry at the FU Berlin
- *Euler-Vorlesung* in Potsdam on 23.5.03
- ECG Workshop at the FU Berlin 25. - 26.6.03
- *31.Berliner Algorithmen Tag* at the HU Berlin on 11.7.03

This summer I will make a research visit at the University of Calgary, Canada.

# References


[AG95] Helmut Alt and Michael Godau. Computing the Fréchet distance between two polygonal curves. *Internat. J. Comput. Geom. Appl.*, 5:75–91, 1995.

[AG99] Helmut Alt and Leonidas Guibas. Discrete geometric shapes: Matching, interpolation, and approximation. In Jörg-Rüdiger Sack and Jorge Urrutia, editors, *Handbook of Computational Geometry*, pages 121 – 153. Elsevier Science Publishers B.V. North-Holland, Amsterdam, 1999.

[AHPMW02] Pankaj K. Agarwal, Sariel Har-Peled, Nabil H. Mustafa, and Yusu Wang. Near-linear time algorithms for curve simplification in two and three dimensions. In Rolf H. Möhring and Rajeev Raman, editors, *Proc. 10th Annual European Symposium on Algorithms (ESA '02.), Rome*, volume 2461 of *Lecture Notes in Computer Science*. Springer-Verlag, 2002.

[God98] Michael Godau. *On the complexity of measuring the similarity between geometric objects in higher dimensions*. PhD thesis, Freie Universität Berlin, Germany, 1998.