Semester Report SS03 of Shi Lingsheng

Name:	Shi Lingsheng
Supervisor:	Prof. Dr. Hans Jürgen Prömel
Field of Research:	Discrete Mathematics
Topic:	Ramsey Theory
PhD Student	at the program since July 2000

Field of Research

In topological Ramsey theory, compactness plays a central role and always comes from the finiteness of partitions in Ramsey theory. Actually, compact spaces have a character of possessing the Ramsey property because of the less obvious connection between Ramsey theory and ultrafilters noted by Hindman, and the relation between ultrafilters and compactness. Kojman [3] recently observed a topological converse of Hindman's theorem and then introduced the so-called Hindman spaces and van der Waerden spaces [4] (both of which are stronger than sequentially compact spaces) corresponding respectively to Hindman's theorem and van der Waerden's theorem (two classic theorems in Ramsey theory). The study of various "Ramsey spaces" is now active.

Results

In 1978, Fürstenburg and Weiss [2] extended Hindman's theorem to a statement implying that compact metric spaces are Hindman and used it to study the phenomenon of uniformly recurrence in compact dynamical systems. Kojman further extended this from compact metric spaces to first countable compact spaces by proving that if the closure of every countable set of a space X is compact and first countable then X is Hindman and van der Waerden. It is clear from definitions that Hindman spaces and van der Waerden spaces are both sequentially compact. But sequentially compact spaces need be neither Hindman nor van der Waerden. In fact, Kojman constructed some spaces that are compact Hausdorff, sequentially compact, separable and first countable at all points but one, but neither Hindman nor van der Waerden. He also proved that the product of two Hindman (resp. van der Waerden) spaces is Hindman (resp. van der Waerden). His proof for Hindman spaces is somewhat complex. I simplified the proof of this result in the framework of set theory as introduced by Fürstenberg [1]. I also strengthened the topological converse of Hindman's theorem by using canonical Ramsey theorem, and introduced differential compactness that arises naturally in this context and forms a larger class than that of Hindman spaces, and studied its relations to other spaces. Recently, Kojman and Shelah [5] found that Hindman spaces and van der Waerden spaces are not the same under the assumption of continuum hypothesis by constructing a van der Waerden space but not Hindman. By using the similar technique, I extended their result by constructing a compact Hausdorff, seperable, van der Waerden space which is first countable at all points but one, and not differentially compact. These results form the main content of second part of my dissertation [6].

Activities

- Block Course "Applied Network Optimization" at the TU Berlin, March 31-April 11, 2003
- Tag der Informatik, at the HU zu Berlin, May 8, 2003
- Berlin - Poznan Workshop in Berlin, May 30, 2003
- Defence of PhD thesis, at the HU zu Berlin, July 10, 2003
- 31. Berliner Algorithmen Tag, at the HU zu Berlin, July 11, 2003
- Seminar "The strange logic of random graphs" at the HU zu Berlin, Spring Term, 2003
- Lectures and colloquia of the European graduate program "Combinatorics, Geometry, and Computation" Spring Term, 2003
- Seminar "Algorithms and Complexities" at the HU zu Berlin (with a talk on "Ramsey numbers of sparse graphs"), Spring Term, 2003

Preview

Some problems arise naturally for future research.

- Are Hindman spaces van der Waerden?
- Are differentially compact spaces Hindman or van der Waerden?

- Are differentially compact spaces closed under products?

References

[1] H. Fürstenberg, Recurrence in Ergodic Theory and Combinatorial Number Theory, Princeton University Press, Princeton, 1981.

[2] H. Fürstenberg and B. Weiss, *Topological dynamics and combinatorial number theory*, J. Analyse Math. 34 (1978) 61-85.

[3] M. Kojman, *Hindman spaces*, Proceedings of the American Mathematical Society, 130, No. 6, (2002) 1597-1602.

[4] M. Kojman, *Van der Waerden spaces*, Proceedings of the American Mathematical Society, 130, No. 3, (2002) 631-635.

[5] M. Kojman and S. Shelah, *van der Waerden spaces and Hindman spaces are not the same*, Proceedings of the American Mathematical Society, to appear.

[6] Shi L., *Numbers and topologies: two aspects of Ramsey theory*, dissertation, Institut für Informatik, Humboldt-Universität zu Berlin, 2003