Semester Report WS02/03 of Ares Ribó Mor

Supervisor:	Prof. Dr. Günter Rote
Field of Research:	Geometry and Combinatorics
Topic:	Self-Touching Configurations
	Map Foldability and Rigidity Theory
PhD Student	at the program since February 1, 2002

Field of Research

A *linkage* is a graph where edges are *rigid bars* with fixed length and vertices are flexible joints. A planar *configuration* of a linkage in $\mathbb{R}^2$ is a mapping of the vertices to points in $\mathbb{R}^2$ such that bars do not intersect. In a *self-touching configuration* bars are allowed to touch and even lie along each other, but not properly cross. The set of feasible motions can be described by linear equations and inequalities, which are stable at least in some neighbourhood of the self-touching configuration ([1]).

A δ*-perturbation* of a self-touching configuration is a repositioning of the vertices within disks of radius δ consistent with the combinatorial embedding in $\mathbb{R}^2$. In a δ*-perturbation* we allow the length of the bars to change slightly.

Results

This semester I have written a generalisation of the *Maxwell-Cremona theorem* for self-touching configurations. The classic Maxwell-Cremona theorem makes a one-to-one correspondence between the set equilibrium stresses of a planar configuration and the set of three-dimensional polyhedral terrains that project onto it. In this new theorem, we give a nice explicit correspondence, between the set of stresses of a plane self-touching configuration and the set of three-dimensional polyhedral terrains with *jump discontinuities* that project onto it. These jump discontinuities are vertical facets given by the self-touching stresses. We have shown that this correspondence is now unique up to a kernel, this is, a set of non-zero forces which lift to the flat polyhedra.

With this interesting new tool, we want to attack some problems regarding self-touching configurations, for example to see whether a self-touching polygonal chain or an arbitrarily flattened tree can be infinitesimally rigid. The idea is to look at the properties of the three-dimensional polyhedral terrain given by this new correspondence.

One of our main problems is to determine whether for every self-touching configurations and for all $\delta > 0$, there exists a δ-perturbation that is a planar configuration. In the last month, together with Sergio Cabello and Günter Rote, we were able to solve some important questions about this conjecture. Consider a self-touching configuration with no infinitesimally close vertices and no vertices between self-touching parallel lines. Let us call *reduced* a self-touching configuration with this assumptions. We have shown that a reduced self-touching configuration has always a δ-perturbation, and we have an algorithm to draw it. Our key idea is to define a suitable partial order of the vertices which allows to draw the configuration without crossings. We have defined a relation as follows: given two vertices x and y, we say that $x \prec y$ if there exist another vertex z such that x touches the edge yz from below or y touches the edge xz from above (we can assume that there are no vertical edges). We have proved that this relation does not contain a cycle, i.e. it defines an order. From there we can give an elegant proof that shows the existence of a perturbation of such a reduced self-touching configuration, in terms of *stresses*, which are the variables of the dual problem (see my last semester report). This is an existential proof, but we have also an explicit algorithm to draw the perturbation, where we draw the vertices appropriately according to the relation mentioned above.

Now we have an initial perturbation of the reduced self-touching configuration. It seems to us that perturbing the rest of the configuration, this is, other vertices which were inside the same δ-disk or between touching parallel lines, can not be too difficult. We have several ideas of how to do this, but we are still thinking on a correct algorithm.

Activities

- Attended the *Monday Lectures and Colloquia* of the Graduate Program
- Attended the *Mittagsseminar Theoretische Informatik* at Freie Universität Berlin. Presentation of the talks:
  - *Pseudo Approximation Algorithms, with Applications to Optimal Motion Planning*, May 8, 2003
  - *Self-touching Configurations can be perturbed*, July 10, 2003

- Attended several sessions of the seminar *Graphenzeichnen* , by Prof. Dr. Günter Rote at Freie Universität Berlin
- Attendance and member of the local organising committee of the *20th International Symposium on Theoretical Aspects of Computer Science (STACS 2003)*, Freie Universität Berlin, February 27–March 1, 2003
- Block Course *Applied Network Optimization*, Technische Universität Berlin, March 31–April 11, 2003
- Block Course *Geometric Graphs*, by Prof. Dr. Janos Pach, at Universitat Politècnica de Catalunya, Barcelona, Spain, April 22–26, 2003
- Workshop of the project *Effective Computational Geometry for Curves and Surfaces ECG*, Freie Universität Berlin, June 25–26, 2003
- *Berliner Algorithmen-Tag*, at Humboldt Universität Berlin, July 11, 2003
- Subreferee for *ISAAC 2003*

Preview

Next semester I plan to visit the research group of Prof. Dr. Jiri Matousek and Prof. Dr. Kratochvil at Charles University, in Prague, as a part of my long-term stay at a partner university.

There I will attend the *EUROCOMB'03*, in Prague, from September 8th to 12th, and the *Workshop on graph homomorphisms and related topics HOMONOLO'03* in Nova Louka, Czech Republic, from September 15th to 19th.

Also in the following we want to write two papers, one about this generalisation of the Maxwell-Cremona theorem for self-touching configurations and a second one about perturbations of self-touching configurations, together with Sergio Cabello and Günter Rote.

References

[1] R. Connelly, E. Demaine, G. Rote, *Infinitesimally locked self-touching linkages with applications to locked trees*,"Physical Knots: Knotting, Link-

ing, and Folding Geometric Objects in $\mathbb{R}^3$". Contemporary Mathematics 304, American Mathematical Society 2002, 287–311.

[2] H. Crapo, W. Whiteley, *Spaces of stresses, projections and parallel drawings for spherical polyhedra*, Contributions to Algebra and Geometry, Volume 35 (1994), No. 2, 259–281.

[3] J. Richter-Gebert, *Realization spaces of polytopes*, chapter 13. Lecture Notes in Mathematics **1643**, Springer–Verlag Berlin Heidelberg 1996.