Semester Report WS02/03 of Katharina Langkau

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Supervisor:	Prof. Dr. Rolf H. Möhring
Field of Research:	Combinatorial Optimization
Topic:	Flows over Time with Flow-Dependent Transit Times
PhD Student	at the program since October 2000

Field of Research and Results

After almost three years in the CGC graduate program, I am now about to finish my PhD thesis. I am taking the opportunity to summarize the main results of my studies. At the beginning there was the following motivation which originates from applications in road traffic control: Can we map the essential properties of flows arising in real-life applications such as road traffic control into an appropriate network flow model? In particular, we had the following property in mind which road users are facing in everyday road traffic; the amount of time needed to traverse a street increases as the arc becomes more congested.

The focus of my research was on a model that reflects this behavior in the following sense: the travel time that flow needs to traverse an arc only depends on the current inflow rate on that arc. We refer to this model as flows over time with inflow-dependent transit times. In this setting, we have considered various classical network flow problems. In the quickest flow problem, we ask for a flow that sends a given demand d from a source node s to a sink node t within minimal time T. This problem is strongly $\mathcal{NP}$-hard in the setting of inflow-dependent transit times. In particular, we cannot hope for a polynomial time algorithm which solves the quickest s-t-flow problem optimally.

However, we have developed several approximation algorithms for the quickest flow problem. Most of the approximation results use a relaxed model of inflow-dependent transit times. This relaxation relies on an expanded graph with fixed transit times on the arcs. To mimic flow-dependent transit times, every arc of the original graph is replaced by a bunch of parallel arcs with different transit times. This expansion has the following intuitive interpretation in the context of road traffic: every expanded arc represents a

multi-lane road; traffic on the same lane travels with the same fixed speed. A driver can choose a lane when entering the road. In contrast to real-life traffic, the driver must remain on this lane and follow the prescribed speed. Flow-dependent transit times are realized as follows. High-speed lanes have a relatively small capacity. If the capacity of this lane is used up, drivers must enter a lane with low speed.

Using this relaxation, we could derive a constant factor approximation for the quickest s-t-flow problem. Based on a slightly stronger relaxation, we developed a fully polynomial time approximation scheme for the quickest multi-commodity problem. Given $\varepsilon > 0$, we can compute a quickest multi-commodity flow that satisfies the given demands within time $1+\varepsilon$ the optimal time. Most of the above results are contained in [1] and [2].

I want to take the opportunity to thank all members of the graduate program who made this work possible: above all , the former coordinator Bettina Felsner, the present coordinator Andrea Hoffkamp, the speaker of the graduate program Helmut Alt and my PhD advisor Rolf H. Möring. I have enjoyed the last three years very much!

Activities

- Talk on *A Model for Flow-Dependent Transit Times* at the 7th Aussois Workshop on Combinatorial Optimization, Aussois, France, March 10–15, 2003
- Talk on *New Approximation Results for Flow-Dependent Transit Times* at the Monday lecture of the graduate program, ZIB Berlin, May 5, 2003

Literatur

[1] A. Hall, K. Langkau, M. Skutella. *Än FPTAS for Quickest Multicommodity Flows with Inflow-Dependent Transit Times*, in Proceedings of APPROX '03, to appear.

[2] E. Köhler, K. Langkau, M. Skutella. *Time-Expanded Graphs for Flow-Dependent Transit Times*, in Proceedings of the 10th Annual European Symposium on Algorithms (ESA), 2002, pp 599–611, Springer LNCS 2461.