Semester Report SS03 of Martin Kutz

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Topic:	Games on Hypergraphs
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Positional Games

In a *weak positional game*, two players, called *Maker* and *Breaker*, alternately color the vertices of a hypergraph in their respective colors, Maker trying to create a monochromatic edge of his color—a Maker win. Breaker tries to prevent Maker from doing so and thus wins when at the end of the game, when all vertices are colored, no edge is monochromatic of Makers color. (He does not win by coloring one edge in his color, though.) Such games have been studied, for example, in and [2] and [4].

Until now, the complexity of such games (that is, deciding whether a given position can be won) is unknown, while the strong version (in which the player who first completes one edge in his color wins) is known to be PSPACE-hard [6].

I suspected that weak games should be governed by computationally simpler principles and last year the last semester I found out that for the restricted class of almost disjoint rank-3-bounded hypergraphs (each edge has at most three vertices and two different edges have at most one vertex in common) the winner of the game can be determined in polynomial time.

Conway's Angel Problem

We consider the following game with a chess king on an infinite checker board: The king moves across the board, according to the usual chess rules and we want to trap him by destroying squares of the board. Precisely, in each move the king steps onto an undamaged square adjacent to his current position and we in turn delete an arbitrary square from the board (except the one under the king, of course). Can you trap the king, that is, do you have a strategy such that at some point the king cannot move any more; or is he able to run on forever?

Berlekamp showed that you can trap him, but the problem gets amazingly difficult if you increase the power of the king. Conway defined a k-angel to be a "king" that can "fly" in one move to any untouched square at distance at most k from his current position. He then asked whether the opponent—whom he figuratively calls the devil–can catch any angel of arbitrary power [1]. This problem remains unsolved for at least two decades now.

I took a very modest approached to this problem by considering an equivalent formulation where the angel may not "fly" but only "run." In this setting, I investigated angels that run at small fractional speed > 1 and devised devil strategies for such slightly strengthened opponents. The first results in this direction are meant to lead to an understanding of the underlying principles of Conway's original problem.

Current Work

My main efforts are still aimed at developing generalizations of a decomposition lemma for weak positional games that is an essential ingredient for my theorem on rank-3-bounded hypergraphs. A success in this direction might yield polynomial algorithms for a much larger class of hypergraphs.

Besides, I try to improve the results on the "fractional angel problem." It also looks promising to attack Conway's problem in three dimensions, where possible escape strategies should be considerably easier to devise as in the two-dimensional setting.

A Sideline Project: Computational Geometry

During the block course on randomized algorithms at the ETH Zürich, I got involved in the *minimum enclosing ball problem* from computational geometry [7]. Motivated by Bernd Gärtners lectures on this topic, I implemented a novel algorithmic idea; and the first rough version of the resulting program showed surprising good performance.

During last March, I visited Zürich again in order to continue work on that algorithm. Together with Kaspar Fischer and Bernd Gärtner I realized a serious implementation of the algorithm, which turned out to beat several current programs for the problem. I will present the results of our work [3] at this year's *European Symposium on Algorithms (ESA)* in Budapest.

Previous Work: Boolean Matrix Roots

My results on the complexity of Boolean matrix root computation have eventually been accepted for publication. I will present my results [5] at *the Ninth International Computing and Combinatorics Conference (COCOON 2003)* at Big Sky Resort, Montana, USA.

Activities

I visited ETH Zürich for three weeks in March 2003 to work on the smalles-enclosing-ball problem. The other two main events of this semester are still to come: two talks at COCOON 2003 and at ESA 2003.

Literatur

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