Semester Report SS03 of Mihyun Kang

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Field of Research:	Random walks, random discrete structures, probabilistic methods
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Field of Research and Results

Generating unlabeled cubic planar graphs uniformly at random

McKay and Wormald [7] showed how to generate random regular graph of moderate degree uniformly at random in an expected polynomial time. But little is known if we restrict our attention to random regular planar graphs.

In [3] we present an expected polynomial time algorithm to generate unlabeled 3-regular (i.e. cubic) planar graphs uniformly at random. We derive recurrence formulas that exactly count all such graphs on n vertices (for even n), based on a decomposition along the connectivity structure of the graph. The recurrence formulas can be evaluated in polynomial time using dynamic programming and they immediately yield the generation procedures. In principle this approach is similar to the one in [2] but it is not so straightforward because we are working with unlabeled structures.

To destroy symmetry of a unlabeled graph we introduce a root as in [1], which is a distinguished vertex, or a distinguished directed edge, or a distinguished face according to a connectivity of a graph. When we count rooted 3-connected cubic planar graphs, we use the fact that they have exactly one or two embeddings where the root edge is embedded at the outer face. In the first case we say that a graph has a sense-reversing automorphism, which we count by an one-to-one correspondence to some special planar networks. As a final step of enumeration and generation we employ that fact that the dual of a 3-connected cubic planar map is a 3-connected planar triangulation. We thus derive the number of 3-connected planar triangulations with specified number of vertices on the outer face by adapting the idea of Tutte [8]. This is joint work with M. Bodirsky, C. Gröpl.

Efficiency test of random number generators using random walks

Markov chain Monte Carlo simulations usually use long sequences of random numbers generated by deterministic rules, so-called pseudorandom number generators. The efficiency of simulation depends on the convergence rate to the stationary distribution and the quality of random numbers used for simulations. Various methods have been developed to measure the convergence rate to the stationary distribution, but the effect of random numbers has not been much discussed.

In [4] we present several algorithms to test the efficiency of pseudorandom number generators. We consider random walks on finite abelian groups and the probability distributions of the first hitting time from a vertex x to another vertex y, which is the minimum number of steps taken to reach y starting from x. Analogous to [5, 6] we derive the probability generating functions based on discrete Fourier analysis. To empirically test the quality of a pseudorandom number generator we propose to use a random sequence $\{X_j\}_{j=0}^{\infty}$ in $\mathbb{Z}_M$ generated by a pseudorandom number generator to determine the walker's movements in computer experiments. Good pseudorandom number generators make the movements fairly random and the random variable $Z = \frac{\overline{\mu}-\mathbb{E}(T)}{\sigma(T)/\sqrt{N}}$ approximates the standard normal distribution for sufficiently large sample size N, where $\overline{\mu}$ is a sample mean.

Activities

- Forschungsseminar at HU Berlin (with a talk "How to generate a cubic planar graph uniformly at random?")
- Lectures and colloquia of CGC
- Teaching a seminar course on "The strange logic of random graphs" (jointly with M. Bodirsky)
- Joint organization of Berlin-Poznań Seminar, HU Berlin, May 30, 2003
- Visiting Adam Mickiewicz University in Poznań, 30 June - 6 July 2003
- Participating in 31.BAT, HU Berlin, July 11, 2003
- Refree of the journal "Combinatorics, Probability & Computing"

Preview

- Workshop on random graphs, Pohang, July 28 – August 1, 2003
- Random structures and algorithms, Poznań, August 9 – 13, 2003
- Eurocomb'03: European conference on combinatorics, graph theory and applications, Prague, September 8 – 12, 2003
- 10th workshop on graph theory: colourings, independence and domination, Karpacz, September 22 – 26, 2003
- Anual CGC workshop, Neustrelitz, September 29–October 1, 2003

References

[1] M. Bodirsky and M. Kang, Generating outerplanar graphs uniformly at random (presented in ALICE 03), submitted.

[2] M. Bodirsky, C. Gröpl and M. Kang, Generating labeled planar graphs uniformly at random, *Proceedings of ICALP2003: Thirtieth International Colloquium on Automata, Languages and Programming*, LNCS 2719, Springer Verlag, 2003.

[3] M. Bodirsky, C. Gröpl and M. Kang, Generating unlabeled cubic planar graphs uniformly at random, *Proceedings of Eurocomb'03*, to appear.

[4] M. Kang, Efficiency test of pseudorandom number generators using random walks, submitted.

[5] M. Kang, First hitting times of simple random walks on graphs with congestion points, *Int. J. Math. Math. Sci.* **2003 (30)** (2003), 1911-1922.

[6] M. Kang, Random walks on a finite graph with congestion points, *Applied Mathematics and Computation*, to appear.

[7] B. D. McKay and N. C. Wormald, Uniform generation of random regular graphs of moderate degree, *Journal of Algorithms* **11** (1990), 325–338.

[8] W. T. Tutte, A census of planar triangulations, *Canadian Journal of Mathematics* **14** (1962), 21–38.