

Semester Report SS03 of Kevin Buchin

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Field of Research: Computational Geometry
Topic: Space-Filling Curves
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Field of Research

This semester I started my research on *(discrete) space-filling curves*. A discrete space-filling curve (SFC) is a bijective map

$$C : \{1, \dots, n^r\} \rightarrow \{1, \dots, n\}^r.$$

A SFC can be seen as a linear ordering or an one-dimensional indexing scheme of a discrete multi-dimensional space.

In particular, I am interested in *proximity or locality preserving* SFCs, i.e. curves that map nearby points in the multi-dimensional space to nearby points in the one-dimensional space, or vice versa. Locality preserving SFC have numerous applications, for instance storing multidimensional data, performing parallel domain decomposition, or traversing an image plane. My main motivation for examining this kind of curves is that they can be used for developing heuristics for geometric computations. The first locality measure for SFC I looked at is the average logarithmic path length of space-filling curves [1], i. e.

$$R_C^{avg} := \frac{1}{\#Neigh(C)} \sum_{(a,b) \in Neigh(C)} \log_2(|C^{-1}(a) - C^{-1}(b)| + 1),$$

where $Neigh(C) = \{(a, b) \in dom(C)^2 \mid \exists i \forall j b_j - a_j = \delta_{ij}\}$ is the set of ordered pairs of neighboring points.

Besides applications that possibly depend on this measure as binary search on an index range, this measure is interesting because it reflects the structure of *recursive space-filling curves (RSFC)*. These are curves that can be subdivided into 2^r equally sized RSFC down to a size of 1. For logarithmic path-length I showed for the two-dimensional case that the quality of a SFC mainly depends on the subcurves of small size. Therefore, I am currently working on a framework for computing optimal SFC of small size. At this, I am aiming at results for a more general class of measures.

Activities

I presented the talk "Logarithmic Path-Length in Recursive Space-Filling Curves" at the *Noon-Seminar* of the TI-AG and subrefereed for the ISAAC 2003. I attended the following events:

- Lectures and colloquia of the CGC
- *Noon-Seminar* of the TI-AG at the FU Berlin
- Lecture *Computational Geometry* at the FU Berlin by Dr. Christian Knauer
- *Euler Lecture* in Potsdam, 23.5 2003
- *ECG Workshop* at the FU Berlin, 25. and 26.6 2003

This summer I will make a research visit at the University of Calgary, Canada.

References

- [1] Jens-Michael Wierum. Logarithmic path-length in space-filling curves. In *Proceedings of the 14th Canadian Conference on Computational Geometry*, pages 22–26, Lethbridge, August 2002.