Shi Lingsheng

Supervisor:	Prof. Dr. Hans Jürgen Prömel
Field of Research:	Discrete Mathematics
Topic:	Ramsey Theory
PhD Student	at the program since July 2000

Results

The Ramsey number $R(G;k)$ of a graph G is the least integer p so that for all k-colorings of the edges of the complete graph K_p, at least one of the monochromatic subgraphs contains a copy of G. It is known that $R(G;k)$ is exponential in the order of dense graphs G. For example, $2^{n/2} \leq R(K_n;2) < 2^{2n}$. In 1972, Chvátal and Harary [2] proved the general result that $R(G;k) \geq (sk^{|E|-1})^{1/|V|}$ where s is the order of the automorphism group of the graph $G = (V,E)$. So in order to force the Ramsey numbers to linearly grow, the graphs should be relatively sparse and have average degree at most $2\log_k n$. In 1973, Burr and Erdős [1] offered a total of \$25 for the conjecture that *there is a constant $c = c(d) \geq 1$ so that $R(G;2) \leq cn$ for all d-degenerate graphs of order n.* Here a graph is d-degenerate if its subgraphs all have minimum degree at most d. Recently, Kostochka and Rödl [5] proved that the Ramsey number of any d-degenerate graph of order n with maximum degree Δ is bounded by $cn\Delta$. If Δ is not restricted, this gives a polynomial bound $\mathrm{O}(n^2)$ for d-degenerate graphs of order n. Shortly later, Kostochka and Sudakov [6] improved this to the almost linear bound $n^{1+o(1)}$. While by the result of Chvátal and Harary, the Ramsey numbers grow at least polynomially for $\mathrm{O}(\ln n)$-degenerate graphs of order n. It would be nice to know if or not they really grow polynomially. I ensured in [8] that they do grow polynomially for bipartite $\mathrm{O}(\ln n)$-degenerate graphs of order n.

In 1927, van der Waerden [10] proved the conjecture of Baudet that *any map from the natural numbers to a finite set is constant on arbitrarily long arithmetic progressions.* A few years later, Gallai [7] extended this to maps on higher dimensions. Furstenberg [3] extended this again to maps with values in a compact metric space. I again extended this in [9] to maps with ranges in nonmetric spaces. This results a short proof of the theorem of Kojman [4] that *if the closure of every countable set of a space X is compact and first countable then X is van der Waerden.* Here a space is van der Waerden if for any sequence there exists a convergent subsequence whose set of indices contains arbitrarily long arithmetic progressions.

Activities

- Academic visitor in the Department of Statistics, University of Oxford, April to September, 2002
- One-Day Combinatorics Colloquium at the University of Reading, United Kingdom, May 15, 2002
- Spring School on "Approximation Algorithms for Hard Problems" in Chorin, Germany, May 20 to 23, 2002
- First-year Students' Talks on Statistics in the Department of Statistics, University of Oxford, May 24, 2002
- The review meeting of the European graduate program, at the FU, June 24, 2002
- Course "Randomized Algorithms" by Professor Colin J. H. McDiarmid in the Corpus Christi College, University of Oxford, Trinity Term, 2002
- Lecture "Polynomial Invariants of Graphs" by Professor Dominic J. A. Welsh in the Mathematical Institute, University of Oxford, Trinity Term, 2002
- Talk on "Ramsey spaces" in the seminar "Algorithms and Complexities" at the HU
- Seminars "Combinatorial Theory" and "Junior Group Theory" in the Mathematical Institute, University of Oxford, Trinity Term, 2002 (with a talk on "Ramsey spaces")

References

[1] S. A. Burr and P. Erdős, *On the magnitude of generalized Ramsey numbers for graphs*, in "Infinite and Finite Sets," Vol. 1, pp. 214-240, Colloquia Mathematica Societatis János Bolyai, Vol. 10, North-Holland, Amsterdam/London, 1975.

[2] V. Chvátal and F. Harary, *Generalized theory for graphs*, Bull. Amer. Math. Soc. 78 (1972), 423-426.

[3] H. Furstenberg, Recurrence in Ergodic Theory and Combinatorial Number Theory, Princeton University Press, Princeton, 1981.

[4] M. Kojman, *Van der Waerden spaces*, Proceedings of the American Mathematical Society, 130, No. 3, (2002), 631-635.

[5] A. V. Kostochka and V. Rödl, *On graphs with small Ramsey numbers II*, submitted.

[6] A. V. Kostochka and B. Sudakov, *On Ramsey numbers of sparse graphs*, manuscript.

[7] R. Rado, *Verallgemeinerung Eines Satzes von van der Waerden mit Anwendungen auf ein Problem der Zahlentheorie*, Sonderausg. Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Klasse 17 (1933), 1-10.

[8] Shi, L. *Do Ramsey numbers grow polynomially for O(lnn)-degenerate graphs of order n?* draft.

[9] Shi, L. *An extension of van der Waerden's theorem*, draft.

[10] B. L. van der Waerden, *Beweis einer Baudetschen Vermutung*, Nieuw Arch. Wisk. 15 (1927) 212-216.