Ares Ribó Mor

Supervisor:	Prof. Dr. Günter Rote
Field of Research:	Geometry and Combinatorics
Topic	Locked and Unlocked Self-Touching Linkages; Map Foldability and Rigidity Theory
PhD Student	at the program since February 1, 2002

Field of Research

During the first semester in the Program, I have worked on several questions related to locked and unlocked configurations.

I am interested in planar linkages embedded in the plane. A *linkage* is a graph where edges are *rigid bars* with fixed length and vertices are flexible joints. A *configuration* of a linkage in $\mathbb{R}^2$ is a mapping of the vertices to points in $\mathbb{R}^2$. In the plane, the combinatorial planar embedding is specified because this cannot change by a motion that avoids crossings. A linkage is *locked* if the configuration space has multiple connected components within the class of embeddings with the same combinatorial planar embedding. I have learnt basics of rigidity theory and several techniques often used for this kind of problems, like force-directed methods and linear programming, first-order rigidity and equilibrium stresses.

It is known that a polygonal tree can be locked [1], and that a polygonal arc, a polygonal cycle or a disjoint union of nonnested polygonal arcs and cycles is always unlocked [2, 4].

The result about polygonal arcs and polygonal cycles settles the well-studied carpenter's rule conjecture. The proof introduces the notion of *expanding motion*, where the distance between every pair of vertices increases or stays the same during the motion. In order to model the expansive property, the theory of tensegrity frameworks is applied. Frameworks consist of bars of fixed length and *struts* of non-decreasing length between every remaining pair of vertices. This is modelled by a linear program, and its feasibility implies the existence of an *infinitesimal motion*. The *Maxwell-Cremona theorem*, which makes a correspondence between the equilibrium stresses of a planar configuration and the three-dimensional polyhedral graph that projects onto it, gives an optimal solution for the dual problem. Combining the infinitesimal motions, a piecewise-differentiable global motion is obtained.

The key distinction between arcs or cycles and trees is that the former

have maximum degree 2, but a tree may have vertices of higher degree. In fact, a single degree-3 vertex can prevent opening. Many examples are constructed in [3]. We have examined two particular cases: We have shown that every monotone tree is unfoldable, using geometric relations between segments with disjoint interiors in the plane and their properties as a partial order; now I try to find out if this is also true for those trees that can be arbitrarily flattened. In the following I want to study which conditions must be satisfied by a tree for being unlocked.

In a *self-touching configuration* (STC) bars are allowed to touch and even lie along each other, but not properly cross. The set of feasible motions is also described by linear equations and inequalities, which are stable at least in some neighbourhood of the STC (see [3]). A *δ-perturbation* of a STC is a repositioning of the vertices within disks of radius δ consistent with the combinatorial embedding in $\mathbb{R}^2$. A configuration is *strongly locked* if for all ε, there is a $\delta > 0$ such that any δ-perturbation is locked within an ε-neighbourhood.

Locked linkages are often based on approximations to STC. I have used the algorithm proposed in [3] to check if several linkages where locked or unlocked. It consists of modelling the linkage as a small perturbation of a STC and checking by linear programming if this STC is infinitesimally rigid. Duality connects the problem to equilibrium stresses. If the linear problem has only trivial solutions then the STC, and thus the linkage, is strongly locked. A non-trivial solution often gives an opening motion. I have illustrated some of these openings with Cinderella.

I am trying to solve the following conjecture, but I have not succeeded yet: for all STC and for all $\delta > 0$, there is a δ-perturbation that is a configuration without bars crossing. The idea for the proof is again to model the configuration by a system of linear inequalities, and to show that the dual problem has a bounded solution. I treat the dual problem in terms of equilibrium stress and I am working on finding out geometric properties of this stresses. With this purpose, I am studying *contact graphs* and their properties.

I have also learnt about other topics, as map foldability in one and two dimensions, shape matching with respect to the symmetric distance and with a reference point (like the Steiner point), and also about the known arithmetic problem of determine the minimum number of distinct sums and distinct distances determined by n different points in the plane.

Activities

- Lectures and Colloquia of the Graduate Program
- *Mittagsseminar Theoretische Informatik* at FU Berlin
- Lecture *Kombinatorische Optimierung*, by Günter Rote at FU Berlin
- Lecture *Algorithmische Geometrie*, by Helmut Alt at FU Berlin
- Subreferee for the *ESA 2002, 10th Annual European Symposium on Algorithms*
- Spring School of the CGC Graduate Program *Approximation Algorithms for Hard Problems*, Chorin, May 20–23, 2002
- Participation and member of the local organising committee for *18th Annual Symposium on Computational Geometry, SoCG'02*, Barcelona, Spain, June 5–7, 2002
- Poster presentation at the review meeting of the CGC Graduate Program, Berlin, June 24, 2002
- Berliner Algorithmen-Tag, July 19, 2002
- Lecture *Deutsch als Fremdsprache* (Grundstufe I, II and Mittelstufe I), at Hartnackschule and at FU Berlin

Publications

- *Recursively Constructible Families of Graphs*, joint work with Marc Noy. Accepted for publication in a especial edition of Advances in Applied Mathematics devoted to the *Workshop on Tutte Polynomials*, Barcelona, Spain, September 12–15, 2001.

Preview

- I will present my topic to my workgroup at the *Mittagsseminar Theoretische Informatik* at FU Berlin, in September 24, 2002
- Annual Workshop of the CGC Graduate Program, which will be held in Hiddensee, October 9–12, 2002

Literatur

[1] T. Biedl, E. Demaine, M. Demaine, S. Lazard, A. Lubiw, J. O'Rourke, S. Robbins, I. Streinu, G. Toussaint, S. Whitesides, *A note on reconfiguring tree linkages: Trees can lock*, Discrete Applied Mathematics **117** (2002), 293–297.

[2] R. Connelly, E. Demaine, G. Rote, *Straightening polygonal arcs and convexifying polygonal cycles*, to appear in Discrete & Computational Geometry.

[3] R. Connelly, E. Demaine, G. Rote, *Infinitesimally locked Self-Touching Linkages with applications to locked trees*, to appear in "Physical Knotting, Linking, and Unknotting" Proceedings. Contemporary Mathematics, American Mathematical Society 2002.

[4] I. Streinu, *A combinatorial approach to planar non-colliding robot arm motion planning*, Proceedings of the 41st Annual Symposium on Foundations of Computer Science (Redondo Beach, California), November 2000, IEEE Computer Society Press, Washington, D. C. , 2000, pp. 443–453.