Katharina Langkau

Supervisor:	Prof. Dr. Rolf H. Möhring
Field of Research:	Combinatorial Optimization
Topic:	Flows over Time with Flow-Dependent Transit Times
PhD Student	at the program since October 2000

Field of Research

During my 6-month-stay in Budapest as a guest of András Frank at the Eötvös Loránd University, I have been involved in work and projects of the Egerváry Research Group on Combinatorial Optimization. One of the main research interests of this group are connectivity problems of networks. By a connectivity problem, we usually mean a problem concerning paths, flows, circuits, trees, cuts, arborescences, etc.

There are several ways how to define that a graph has a certain connectivity. One main objective is to study these different types of graph connectivity and characterize graphs with respect to its connectivity. Problems arising in this area are, for example, to efficiently decide if a graph has a certain connectivity property. A related problem concerns augmentation of graphs. Here the question is, how many edges one has to add to a given graph in order to guaranty a certain connectivity. Another objective is to find constructive characterizations of graphs with a certain connectivity property. During my stay in Budapest, I learned about these central questions in connectivity theory and about commonly used solution techniques.

Moreover, I worked on a particular problem concerning orientations of graphs. In a graph $G = (V, E)$ (directed graph $D = (V, A)$) the *local edge-connectivity* $\lambda(x, y; G)$ ($\lambda(x, y; D)$) of nodes x and y is the maximum number of edge-disjoint paths connecting x and y (from x to y). Nash-Williams' strong orientation theorem says that every undirected graph $G = (V, E)$ has an orientation $\vec{G}$ for which $\lambda(x, y; \vec{G}) \geq \lfloor \lambda(x, y; G)/2 \rfloor$.

For Eulerian graphs, it is not difficult to see that the theorem holds. Any Eulerian orientation of G will half the undirected local edge-connectivity between any two vertices. Based on this idea, Nash-Williams proved an even stronger result, namely, he could show the existence of a matching M of all odd-degree vertices of G, such that any Eulerian orientation of $G + M$ yields an orientation $\vec{G}$ where $\lambda(x, y; \vec{G}) \geq \lfloor \lambda(x, y; G)/2 \rfloor$. Unfortunately, so far no simple proof of the theorem has been found and its links to other connectivity problems are not very well-understood.

The weaker version of Nash-Williams' theorem says that an undirected graph G has a k-edge-connected orientation iff G is $2k$-edge-connected, where we call a graph $G = (V, E)$ (directed graph $D = (V, A)$) *k-edge-connected* if $\lambda(x, y; G) \geq k$ ($\lambda(x, y; D) \geq k$) for all $x, y \in V$. In this global setting, not only a matching of the odd-degree-vertices can be found, which then yields a good orientation for G as described above. We could prove that one can even choose edge-disjoint paths in G pairing the vertices of odd degree. Moreover, we can show the existence of a k-edge-connected orientation $\vec{G}$ such that these paths become directed paths in $\vec{G}$. By reorienting $\vec{G}$ along any subset of these paths, k-edge-connectivity is preserved. A further goal is to prove that this path-criterion also holds for the strong version of Nash Williams' theorem.

Activities

- 6-month-stay in Budapest as a guest of the Eötvös Loránd University, supervisor András Frank
- Talk at the seminar of the Egerváry Research Group on Combinatorial Optimization, Eötvös Loránd University, Budapest, Hungary, April 15, 2002
- Spring school of the CGC graduate program *Approximation Algorithms for Hard Problems*, Chorin, Germany, May 20 – 23, 2002
- Talk at the review meeting of the CGC graduate program, Berlin, Germany, June 24, 2002
- Lectures and colloquia of the Egerváry Research Group on Combinatorial Optimization, Eötvös University, Budapest, Hungary

Publications

- Ekkehard Köhler, Katharina Langkau, Martin Skutella,
  Time-Expanded Graphs with Flow-Dependent Transit Times,
  in Rolf H. Möhring and Rajeev Raman (eds.): Algorithms - ESA '02, Lecture Notes in Computer Science 2461, Springer: Berlin, 2002,

599-611, Proceedings of the 10th Annual European Symposium on Algorithms (ESA'02).

Literatur

[1] C.St.J.A. Nash-Williams. On orientations, connectivity and odd vertex pairings in finite graphs, *Canad. J. Math.* **12** (1960) 555-567.