Semester Report SS02 of Martin Kutz

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Field of Research:	Algorithmic Graph Theory, Combinatorial Game Theory
Topic:	Games on Hypergraphs
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Previous Work and Results

Until the end of 2001, I mainly worked on three different topics. One lead to a conference talk, the other two resulted in articles that are now accepted for journal publication.

Treeifying posets Manuel Bodirsky, also PhD student of the program, had transformed a variant of an open question from computer linguistics into a graph theoretic setting: *When can a partial order be extended to a tree, while certain given pairs of elements remain incomparable?* Together we developed an algorithm that constructs a solution in quadratic time, provided one exists. I presented our results [1] at the 19th International Symposium on Theoretical Aspects of Computer Science (STACS), March 2002.

Lucas chains Already before I had come to Berlin, I had written up the main results of my diploma thesis [7] about Lucas chains for journal publication. Lucas chains are a variant of addition chains [5]. They were first considered by Montgomery [8] and later rediscovered for public-key cryptography [10]. My manuscript [6] is now accepted for publication.

Light switch configurations This little sideline project was launched by Dennis Epple, student at the Freie Universität Berlin, as a coffee break math problem: *There is a standard setup for operating a single electric light from several switches. Is it best possible?* We developed a graph theoretic model for such electric light switch circuits and investigated this question in greater depth. To our surprise, we were actually able to prove a sharp lower bound on the number of elementary devices in such circuits and could thus answer that question in the affirmative. I wrote up our result and the paper [2] has just been accepted for publication.

Current Research

This semester I obtained a first result about my main subject, *games on hypergraphs*. However, I also spent much time on the investigation of digraph powers and finally succeeded to determine the complexity of Boolean-matrix-root computation—a long standing open question.

Game theory I am investigating the structure of biased positional games on hypergraphs, considered for example in [3]. In such a game, two players, Black and White, alternately color the vertices of a hypergraph in their respective colors. Black wins if he manages to color a complete hyperedge in his color. White wins if he prevents Black from doing so. (He does not win by coloring one edge completely white; hence the term *biased*.)

Until now, nothing was known about the complexity of such games. Though the unbiased version (the player who first completes one edge in his color wins) is known to be PSPACE-hard [9], I suspected that unbiased games should be governed by computationally simpler principles. In fact, I was able to show that for almost disjoint 3-uniform hypergraphs (each edge has exactly three vertices and two different edges have at most one vertex in common) the winner of the game can be determined in polynomial time.

My goal now is to get rid of the almost-disjointness condition and then to generalize the result to arbitrary hypergraphs. Perhaps one cannot maintain deterministic solvability but it might be possible to show that the set of winning positions lies in NP.

Boolean matrix roots Multiplication of Boolean 0-1-matrices is defined as ordinary matrix multiplication with $+$ and $\times$ replaced by the Boolean operations $\vee$ and $\wedge$. A kth root of a Boolean $n \times n$-matrix A, $k \geq 2$, is another Boolean matrix R with $R^k = A$.

I showed the problem of determining whether a given Boolean matrix has a kth root NP-complete for every single parameter $k \geq 2$. This solves a question that had been open for twenty years [4, p. 255].

Further, I investigated a subclass of Boolean matrices for which root finding turns out graph-isomorphism complete. This may come quite as a surprise since all problems that are known to lie in this complexity class are more or less obviously "isomorphism problems" of certain combinatorial structures.

Activities

- Attended the Fall School "Discrete geometry—triangulations from various points of view", October 4 to 6, 2001 in Alt Ruppin.
- Talk on "Treeifying posets with incomparability constraints" in the colloquium of the graduate program on January 28, 2002.
- Talk on "Pure dominance constraints" at the 19th International Symposium on Theoretical Aspects of Computer Science (STACS), March 14 to 16, 2002 in Antibes Juan-les-Pins, France.
- Attended the Spring School "Approximation algorithms for hard problems", May 20 to 23, 2002 in Chorin.

Preview

I will attend the CGC workshop on the island of Hiddensee in October, where I will also give a talk about my work on Boolean matrix roots. After that I will go to Prague for my half-year stay at Charles University. That period will be interrupted for five weeks when I visit ETH Zurich for two Block courses on randomized algorithms and on approximation.

References

[1] Manuel Bodirsky and Martin Kutz. Pure dominance constraints. In *Proceedings of the 19th Annual Symposium on Theoretical Aspects of Computer Science (STACS)*, pages 287–298, 2002.

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[3] A.W. Hales and R.I. Jewett. Regularity and positional games. *Transactions of the American Mathematical Society*, 106:222–229, 1963.

[4] Ki Hang Kim. *Boolean matrix theory and applications.* Marcel Decker, Inc., 1982.

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[6] Martin Kutz. Lower bounds for lucas chains. To appear.

[7] Martin Kutz. Grundlegende Betrachtungen zu einer Variante von Additionsketten. Master's thesis, Rheinische Friedrich-Wilhelms-Universität Bonn, Germany, 2000.

[8] Peter L. Montgomery. Evaluating recurrences of form $X_{m+n} = f(X_m, X_n, X_{m-n})$ via Lucas chains. unpublished, `ftp.cwi.nl: /pub/pmontgom/Lucas.ps.gz`.

[9] Stephan Reisch. Gobang ist PSPACE-vollständig. *Acta Informatica*, 13:59–66, 1980.

[10] Sung-Ming Yen and Chi-Sung Laih. Fast algorithms for LUC digital signature computation. In *IEE Proceedings—Computers and Digital Techniques*, volume 142 (2), pages 165–169, March 1995.