

Vanessa Kääh

Supervisor: Prof. Dr. Rolf H. Möhring
Field of Research: Scheduling
Topic: Scheduling with AND/OR-networks
PhD Student: at the program since March 2000

Field of Research and Results

Since I returned from Zurich, I have again concentrated on my main field of research, scheduling with AND/OR-networks. The problem is to find a schedule for a set of jobs restricted by AND/OR precedence constraints and optimizing some objective function. The AND/OR-networks are a useful generalization of the classical AND-networks which model precedence constraints of the form that a job has to wait for the completion of all its direct predecessors. In contrast to this, an OR-node only has to wait for the completion of at least one of its direct predecessors. In some applications one even considers the case where only one predecessor of an OR-node has to be processed at all and the others may be left unprocessed. According to the different applications, the aim of scheduling with AND/OR-networks is to find a feasible schedule optimizing a given objective function, such as the total completion time, or the weighted sum of completion times, or minimum lateness. Most of the time I have focused on the total completion time. The classical questions about feasibility, transitivity, and earliest start schedules of AND/OR-networks have been studied in [?] for example.

In continuation of previous work, I have examined the notion of criticality in AND/OR-networks. It was possible to identify path- and cut-critical jobs and sets, reflecting the idea of longest paths and cuts through longest paths in classical networks. The system of path- and cut-critical sets form a pair of blocking clutters. As a symmetric counterpart to these two definitions it is possible to educe delay- and trunk-critical jobs and sets, again, their systems form a pair of blocking clutters. The delay-critical jobs are characterized by the “delaying” property of a critical job, i.e. a job on a longest path, in standard networks. By the blocking property of the two pairs of systems of critical sets one can conclude that for a job j of an AND/OR-network, j is path-critical iff j is cut-critical, and j is trunk-critical iff j is delay-critical. In general, neither j path-critical implies j trunk-critical nor the other way around. If the AND/OR-network forms an in-tree, the definitions

are equivalent, but weaker properties such as N-freeness of the partial order do not suffice for the equivalence. In contrast to N-freeness, the notion of series-parallel partial orders does not extend in a straight forward manner to AND/OR-networks.

If restricted to the pure structural properties of critical sets, it is possible to interpret them as monotone boolean functions. For monotone boolean functions it is known that they can be computed over $\{\vee, \wedge\}$ -circuits, which are very similar to our AND/OR-networks. In fact, the AND/OR-network can be interpreted in four different ways, yielding the circuits of four different boolean functions. For monotone boolean functions, it is also known, that the problem of finding an assignment with a minimum number of ones is solvable in polynomial time, corresponding to our result, that finding any critical set in an AND/OR-network is easy. In contrast to this, the question of deciding whether a certain variable belongs to such a minimal set of ones is NP-complete. I have proved earlier that deciding whether a job j is critical in any of the four meanings is NP-complete. This question can be reduced to the question of membership in a minimal set of ones for a variable, thus giving the mentioned hardness result.

Inspired by another idea of scheduling with standard networks I have examined critical sets with respect to their usefulness in defining neighborhood moves in a neighborhood search. In standard networks there are several results concerning feasibility of neighbors when reorienting edges on longest paths. Unfortunately one cannot guaranty for feasibility when changing the waiting job of an OR-node contained in a path- or trunk-critical set. On the other hand, by the definition of trunk-critical sets, it seems auspicious to exchange waiting jobs of OR-nodes contained in a trunk-critical set, which might lead to a good heuristic.

Activities

- Lectures and Colloquia of the Graduate Program
- Member of the selection committee for a C3 professorship in discrete mathematics at the TU Berlin
- CGC Spring School *Approximation Algorithms for Hard Problems* in Chorin, May 20 – 23, 2002

- Poster presentation on the CGC review meeting at FU Berlin, June 24, 2002
- Support of a students talk in the block seminar *Network Optimization* by Rolf H. Möhring in Warnemünde, June 28 – 30, 2002
- Seminar talk *Philip Klein: Preprocessing an undirected planar network to enable fast approximate distance queries* at TU Berlin, July 5, 2002

Preview

- CGC Fall School *Algorithms for Hard Problems* in the Bildungszentrum Matt, Schwarzenberg, Switzerland, September 23 - 27, 2002
- CGC general workshop on Hiddensee, October 9 – 12, 2002

Literatur

[MSS00] R. H. Möhring, M. Skutella, and F. Stork. Scheduling with AND/OR-precedence constraints. *Technical Report 689-2000*, Technische Universität Berlin, 2000.