Hendricus van der Holst

Supervisor:	Prof. dr. Martin Aigner
Field of Research:	Graph theory and Combinatorics
Topic	graph polynomials and topological graph theory
Postdoc Student	at the program since November 1 2000

Field of Research and results

This semester we continued working on the graph polynomial introduced by Arratia, Bollobás, and Sorkin [1]; see last semester report. We discovered some new results, and a paper on this topic has been submitted.

One of the problems I am working on at the moment is to get a polynomial time algorithm for calculating the interlace polynomial for isotropic systems of bounded branch-width. Branch-width has been defined for graphs and matroids; for graphs it can be shown that the branch-width of the graph is small if and only if the tree-width of the graphs is small. It is straightfoward to define branch-width for isotropic systems. A branch-decomposition of an isotropic system with n element consists of a ternary tree T (a tree in which each vertex has degree three or one) with exactly n leaves and a labelling of these leaves with the element of the isotropic system. For each edge e in T, $T \setminus e$ has exactly two components, and hence for each edge e of T, the set of elements of the isotropic system is partitioned into two sets X and Y, the elements corresponding to the leaves of one component and the elements corresponding to the leaves of the other component. Every partition (X, Y) of the set of elements of the isotropic system defines a separation of a certain order. So each edge of T defines a separation. The width of the branch-decomposition is defined as the maximum order over all these separations. The branch-width is defined as the minimum width over all branch-decompositions. It might also be interesting to characterize the isotropic systems that have branch-width bounded by small constant, say two or three.

I finally succeeded in making a new polynomial time algorithm for testing whether a graph has a linkless embedding or not. A linkless embedding of a graph G is an embedding of G into 3-space such that the images of every two disjoint circuits have zero linking number (taken over $\mathbb{Z}_2$). The first polynomial time algorithm was given by Robertson and Seymour, and is a consequence of their Graph Minors project. This is however a purely existence result. Later Robertson, Seymour, and Thomas [2] gave the excluded minors for the class of graphs that have a linkless embedding, making the algorithm explicit. However, their algorithm is more a theoretical result than that it can be used in pratice. My algorithm seems to be more practical.

Activities

I am currently at Oxford University. On 27 June I gave a talk about the interlace polynomial at the Combinatorial Theory Seminar, Oxford.

Literatur

[1] R. Arratia, B. Bollobás, and G.B. Sorkin, *The Interlace Polynomial: A New Graph Polynomial*, Proceedings of the Eleventh Annual ACM-SIAM Symposium on Discrete Algorithm, 237–245, San Francisco, California, 2000.

[2] N. Robertson, P. Seymour, and R. Thomas, *Sach's Linkless Embedding Conjecture*, Journal of Comb. Theory, Series B, **64** (1995), 185–227.