Semester Report Martin Thimm

Name:	Martin Thimm
Supervisior	Prof. Dr. Hans Jürgen Prömel
Field of research	Approximation algorithms and nonapproximability
Topic:	Expander graphs
PhD Student	
at the program since	April '99

Fields of research

I mainly work on approximation algorithms and nonapproximability. Expander graphs are "sparse" graphs with "high" connectivity and have various applications in these areas. They can be used for both lower bound proofs (e.g. [5]) and also upper bounds (e.g. [1]). The existence of the desired expanders can be shown by probabilistic arguments (e.g.[8]), but very little is known about how to construct them deterministically [6, 7, 11].

Results

One example for the use of expanders are nonapproximability results. We improved the best known lower bound for the Steiner Tree Problem – this follows from a nonapproximability result for *VERTEX-COVER* in graphs of bounded degree [5] – by a factor of 3. This result [10] will be presented at MFCS2001.

Expanders

What makes a graph a good expander? How difficult is it to approximately calculate the expansion properties of a graph?

These are the questions I'm working on. It is known to be hard to calculate expansion of a given graph exactly [4]. One (only) knows that expansion is closely related to the second largest eigenvalue of the adjacency matrix of the graph [2, 3, 9]. The expanders constructed in [7] have an optimal second largest eigenvalue, but probabilistic arguments prove that there exist constructions with even better expansion.

Therefore, our approach is different. We try to prove expansion properties directly without using eigenvalues. One problem with this is that there is no general framework to do this.

Since we do not use randomness, all the constructions we use are in a sense regular. This regularity is necessary to efficiently construct the graphs and to analyze them. On the other hand it often produces some "hidden" regularity which was not intended and which destroys the expansion property.

One approach to overcome this difficulty is to use the same kind of regularity two times (in a nested way) with in some sense different signs. What also seems helpful is to use regularities which produce unbalanced structures.

Using these two ideas we constructed two families of very sparse graphs (maximum degree 4). Fibonacci numbers are used to define the construction which leads to both regularity and unbalanced behaviour. Empirically these graphs are higly expanding (we are able to test this with some simple but useful heuristcs up to a few thousend nodes).

Activities

- Lectures and Colloquium of the CGC program
- Berlin-Poznan-Seminar, HU Berlin, 23.-24.03.01
- Research seminar "algorithms and complexity" at HU with talks "Generalized Submodular Cover Problems and Applications" and "Undirected Connectivity in $O(\log^{1.5} n)$ Space"

Preview

The above mentioned ideas which led to our construction have now to be formalized to really prove the expansion property of these graphs.

Literatur

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