Semester Report Shi Lingsheng

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Field of Research:	Discrete Mathematics
Topic:	Ramsey Theory
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Field of Research

The classic theorems of Ramsey theory are known to many mathematicians for elegance in their statement. Van der Waerden: If the natural numbers are finitely colored then one color class contains arithmetic progressions of arbitrary length. Schur: If the natural numbers are finitely colored then one color class contains l, m and n with $l + m = n$. Ramsey: If a gaph contains sufficiently many vertices (dependent on k) then it must contain either a clique or a stable set of vertices of size k.

The origins of Ramsey theory are diffuse. Frank Ramsey was interested in decision procedures for logical systems. Issac Schur wanted to solve Fermat's last theorem over finite fields. B. L. van der Waerden solved an amusing problem-and immediately retured to his researches in algebraic geometry. The emergence of Ramsey theory as a cohesive subdiscipline of combinatorics occurred in 1970s. The field is now alive and exciting.

Results

Among the most interesting problems in graph Ramsey theory are the linear bounds for graphs with certain upper bound constraints on the average degree. A set of graphs has linear Ramsey numbers if there exists a constant c so that whenever we bicolor the edges of a complete graph of order cn, we find a monochromatic copy of any graph of order n in this set. In 1973, Burr and Erdős offered a total of $25 for the following conjecture [1].

Any set of graphs whose subgraphs all have bounded average degree has linear Ramsey numbers.

Some weakened versions of this conjecture were obtained in the last two decades. In 1983, Chvátal, Rödl, Szemerédi, and Trotter [3] showed that any set of graphs with bounded maximum degree has linear Ramsey numbers. In 1993, Chen and Schelp [2] extended the result of Chvátal et al. by replacing the bounded maximum degree condition by the following weaker one. A graph of order n is p-arrangeable if its vertices can be ordered $v_1, v_2, ..., v_n$ so that for each integer i with $1 \leq i \leq n$, at most p vertices among $\{v_1, v_2, ..., v_i\}$ have a neighbor $v \in \{v_{i+1}, v_{i+2}, ..., v_n\}$ adjacent to v_i. They proved that any set of p-arrangeable graphs has linear Ramsey numbers. They also showed that a planar graph is 761-arrangeable, which was later improved to 10-arrangeable by Kierstead and Trotter [4]. Thus their results imply that the set of planar graphs has linear Ramsey numbers. Recently, Rödl and Thomas [5] proved that any set of graphs without a fixed topological clique has linear Ramsey numbers.

Given a tree T of order p, denote by $\mathcal{T}_p$ the set of all trees obtained by replacing some edges of T by separate two-paths between their ends. Using a similar method of Rödl and Thomas, I proved in [6] that for each integer $p > 2$, if a graph is not $4p^4$-arrangeable then it contains a copy of some tree in $\mathcal{T}_p$. By the result of Chen and Schelp this implies that a set of graphs without the copy of any tree in $\mathcal{T}_p$ has linear Ramsey numbers and by a result of Kierstead and Trotter it implies that such graphs have bounded "game chromatic number".

In 1973, Burr and Erdős [1] also offered a total of $25 for deciding whether *the set of cubes has linear Ramsey numbers.* Though I cannot settle this question, I have deduced in [7] a polynomial bound $R(Q_n) < 2^{(3+\sqrt{5})n/2+o(n)}$ by showing that for any positive constant c and bipartite graph $G = (U, V; E)$ of order n where the maximum degree of vertices in U is at most $c \log n$, $R(G) < n^{1+(c+\sqrt{c^2+4c})/2+o(1)}$ based on an idea of Kostochka and Rödl. It improves the old bound 2^{cn^2} due to Beck and the bound $2^{cn \log n}$ due to Graham, Rödl and Ruciński.

Activities

- 26. Berlin Algorithm Day, February 16, 2001
- ADiMMO - Workshop in Trier, Germany, March 15 - 16, 2001 (with talk on "A bound for Ramsey numbers")
- Berlin - Poznan Workshop in Berlin, Germany, March 23 - 24, 2001 (with talk on "Cube Rasmey numbers are polynomial")
- Block - Course "Connectivity Problems of Networks: Structures and Algorithms" in Berlin, Germany, April 1 - 12, 2001

- Block - Seminar "List - Decoding" in Berlin, Germany, April 20, 2001 (with talk on "Chinese remaindering with errors")
- Workshop on Combinatorics, Geometry, and Computation in Monte Verità, Centro Stefano Franscini, Ascona, Ticino, Switzerland, May 13 - 15, 2001 (with talk on "A sequence of formulas for Ramsey numbers")
- Workshop on Combinatorics and Random Structures in Berlin, Germany, July 5 - 7, 2001
- 27. Berlin Algorithm Day, July 6, 2001
- Seminar "Algorithms and Complexities" at the HU (with talk on "Arrangeability and tree subdivisions")
- Lectures and colloquium of the European graduate program (with talk on "Cube Ramsey numbers are polynomial")

References

[1] S. A. Burr and P. Erdös, *On the magnitude of generalized Ramsey numbers for graphs*, in "Infinite and Finite Sets," Vol. 1, pp. 214-240, Colloquia Mathematica Societatis János Bolyai, Vol. 10, North-Holland, Amsterdam/London, 1975.

[2] G. Chen and R. Schelp, *Graphs with linearly bounded Ramsey numbers*, J. Combin. Th. (B) 57 (1993), 138-149.

[3] V. Chvatál, V. Rödl, E. Szemerédi and W. T. Trotter, Jr., *The Ramsey number of a graph with bounded maximum degree*, J. Combin. Th. (B) 34 (1983), 239-243.

[4] H. A. Kierstead and W. T. Trotter, *Planar graph coloring with an uncooperative partner*, DIMACS Series in Discrete Math. and Theor. Comp. Sci. 9 (1993), 85-93.

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[6] Shi Lingsheng, *Arrangeability and trees*, script.

[7] Shi Lingsheng, *Cube Ramsey numbers are polynomial*, Random Structures & Algorithms, to appear.