Semester Report Julian Pfeifle

Supervisor:	Prof. Günter M. Ziegler
Field of Research:	Discrete and Combinatorial Geometry
Topic:	Triangulated Complexes
PhD Student	at the program since March 1, 2000

Field of Research and Results

This semester was dedicated to my "CGC-Auslandssemester" at the Department of Theoretical Computer Science at ETH Zürich, where I collaborated with members of the research group under Prof. Emo Welzl. My main area of research there were *unique sink orientations* of d-cubes, which are orientations of the edges of a combinatorial d-cube such that every face contains a unique sink—whence the name. Perhaps the main reason for their importance lies in the fact that they provide a common combinatorial abstraction for various geometric optimization problems, such as linear programming (they generalize the concept of *abstract objective functions*), *LP-type problems* (such as finding the minimum enclosing ball of n points in d-space), and *linear complementarity problems*. In collaboration with Ingo Schurr and Uli Wagner, I focused on structural properties of the set of all unique sink orientations (USOs) on a d-cube, such as the question whether in any such USO one can always reverse the orientation of d edges such that the resulting total orientation still has the unique sink property; whether one can get from any USO to any other by successive "edge flips"; whether there exists any polytope that encodes unique sink orientations in analogy to the way the "secondary polytope" of Gelfand, Kapranov, and Zelevinsky [1] encodes triangulations of point configurations; etc. Before and at the CGC-workshop "Towards the Peak" below the Gotthard pass in Switzerland, I investigated *oriented matroid fans*, a concept that arose in generalizing unique sink orientations that stem from linear complementarity problems.

Also during my stay at ETH Zürich, I thought about some questions related to cubical polytopes, in particular whether they can be reconstructed from their dual graph, but so far without definite results.

Continuing work from last semester on Kalai polytopes, and following a suggestion of Günter M. Ziegler, I was able to shorten considerably

a proof by Robert Hebble and Carl Lee [2] that the dual graphs of Kalai's squeezed 2- and 3-spheres are Hamiltonian.

Activities

- Attendance of the lectures and colloquia of the CGC
- March 12–July 31, 2001: "CGC-Auslandssemester" at the Institute of Theoretical Computer Science at ETH Zürich, with the group of Emo Welzl
- Lecture *"Geometric Arrangements and their Applications"* by Micha Sharir at ETH Zürich
- Attendance of the *Mittagsseminar* of the Theoretical Computer Science group at ETH Zürich
- Attendance of the *17th European Workshop on Computational Geometry*, March 26–28, 2001, in Berlin
- Presentation of the talk *All of Kalai's "Many" Squeezed 3-Spheres are Polytopal* at the *CGC-Workshop on Geometry, Combinatorics, and Computation*, May 13–15, 2001, at Monte Verità, Ascona
- Presentation of the talk *¿Cuántas 3-esferas hay?* at the *IX Encuentros de Geometría Computacional*, July 2–4, 2001, in Girona
- Attendance of the CGC-workshop *Towards the Peak*, August 24–30, 2001, underneath the Gotthard pass in Switzerland

Preview

On the cubical side of things, I plan to continue thinking about whether cubical polytopes can be reconstructed from their dual graphs, and about various constructions related to them. Also I would like to know how many unique sink orientations come from oriented matroid fans—all 3-dimensional ones do.

A new perspective that opened up during the *Towards the Peak* workshop is finding tight upper bounds on the number of pivot steps that a simplex algorithm takes on simple 4-polytopes—in particular, are there simple 4-polytopes on which the RANDOM-EDGE rule is superlinear?

References

[1] I.M. Gelfand, M.M. Kapranov, A.V. Zelevinsky, *Discriminants, resultants, and multidimensional determinants*, Birkhäuser, 1994

[2] Robert Hebble and Carl Lee, *Squeezed 2-Spheres and 3-Spheres are Hamiltonian*, preprint 2000, `http://www.ms.uky.edu/~lee/ham.pdf`