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Field of Research:	Combinatorial Optimization
Topic:	Traffic Networks and Dynamic Flows
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Field of Research

The problem I am working on arises in the area of traffic routing. Studies show that traffic will increase dramatically in the next couple of years causing even more traffic jams on the roads than we have already today. To cope with the problems that are brought about by the much higher volume of traffic, more and more vehicles will get equipped with so-called route guidance systems. They guide the driver by visual and acoustic indicators to the destination, which has been entered at the beginning of a trip. The routing algorithms used by such systems are fairly simple. Given a request, they compute the shortest path from the starting point to the destination, possibly using some information on the current traffic situation. The algorithms do not take into account the effect of their own route recommendation. In view of the fact that route guidance systems will be more widely used in the future, a more sophisticated approach is needed.

I am particularly interested in finding a realistic and at the same time tractable model to describe traffic. One approach is to model traffic as a network flow. The travel time along an arc is then assumed to be a monotonically inreasing function of the total flow on that arc. We use arc capacities to bound the amount of arc flow. Given the starting points and destinations of the drivers, the goal is to meet all the requests while minimizing total travel time. This leads to a minimum cost multicommodity flow problem. Such a *static* approach is described in detail in [1]. The drawback of this static flow model is that a path flow is considered to be "simultaneous" on all edges of the path; it is not "moving" through the network with progressing time.

The notion of dynamic flows was introduced by Ford and Fulkerson [2]. Given a network $(G = (V, A), u, s, t)$ and travel times τ_a on the arcs, a mapping $f : A \times \{0, \ldots, T\} \to \mathbb{R}_0^+$ is said to be a *feasible dynamic flow* if

$$\sum_{a \in \delta^-(v)} f_a(\theta - \tau_a) - \sum_{a \in \delta^+(v)} f_a(\theta) = 0 \qquad \forall v \notin \{s, t\},\ 0 \le \theta \le T$$

$$0 \le f_a(\theta) \le u_a \qquad \forall a \in A,\ 0 \le \theta \le T$$

The interpretation of a dynamic flow f is that $f_a(\tau)$ units of flow are entering the arc a at time τ. A helpful tool to understand dynamic flows is the so-called *time-expanded graph* $G_T = (V_T, A_T)$, which is derived from the original graph $G = (V, A)$ by replacing each vertex $v \in V$ by T copies $v_1, \ldots, v_T$. For each arc $a = (v, w)$ in the original arc set A, we add arcs $(v_\theta, w_{\theta+\tau_a}), \theta = 1, \ldots T - \tau_a$ to the expanded graph. A small example of a time-expanded graph is given below. A dynamic flow then simply relates to a static flow in the time-expanded graph.

time-expansion.eps

In the definition of a dynamic flow the travel times τ_a for an arc $a \in A$ are given *in advance* and that is essential for the construction of the time-expanded graph. But in order to model traffic more realistically, these travel times should *depend on the flow*, which makes the modelling much more complicated. A good model should capture both, flow-dependent travel times and the notion of a dynamic flow. One idea to design such a model is to consider a slightly more complicated time-expanded graph, which indirectly incorporates the fact that arc travel times depend on the arc flow. In this model an arc a is not only copied once for each time period with a fixed travel time τ_a. Instead we allow different travel times for each time period, whose usage is regulated by arc capacities. Consider the following simple example:

`time-cap-exp.eps`

In this example a maximum of u_1 units of flow can use the fast arc with travel time one. If more flow is entering that arc at the same time $t = 0$, the additional flow will have to use slower arcs. In the example we allow u_2 units of flow to use the arc with travel time 3, and u_3 units to use travel time 6. That way we indirectly obtain travel times that depend on the flow.

During the last months I mainly worked with this new model and could show some first results, among them some NP – completeness proofs and an approximation for the single source – single sink case with geometrically increasing capacities. The rich structure of this graph will keep me busy for some more time. An implementation of this model is planned for the coming semester.

Preview

- Workshop of the Munich Graduate Program "Angewandte Algorithmische Mathematik" on *Analysis and Optimization* in Munich, Germany, October 8 – 10, 2001.
- Block courses on *Randomized Algorithms* and on *Topological Methods in Combinatorics and Geometry* in Zurich, Switzerland, October 22 – November 23, 2001.

Literatur

[1] Olaf Jahn. *Multicommodity Flow-Modelle und Algorithmen zur dynamischen Lenkung von Verkehrsströmen*, Diplomarbeit, Technische Universität Berlin, 1998.

[2] L.R. Ford, D.R. Fulkerson. *Flows in Networks*, Princeton University Press, New Jersey, 1962.