Semester Report Carsten Lange

Supervisor:	Prof. Günter M. Ziegler
Field of Research:	Discrete and Combinatorial Geometry
Topic:	Differential Geometry and Combinatorics
PhD student	at the program since January 1, 2000

Field of Research and Results. To derive and generalise the notion of curvature for CW-complexes introduced by Robin Forman, [For99, For00], I defined a weighted difference operator, [Lan01]. This operator as well as the curvature term derived from it fits into the picture one has from differential geometry: There are combinatorial formulae of Weitzenböck type which relate the two Laplacians defined by the difference operator and the boundary and coboundary maps to this curvature. Moreover, in the special case of closed cellular surfaces and a certain class of weights, it is possible to derive a combinatorial version of the classical theorem of Gauß and Bonnet: The sum over the Ricci curvature of every edge is a multiple of the Euler number of the surface.

The following result for 3-dimensional reflexive lattice polytopes P which contain the origin as only lattice point in their interior is known:

$$\sum_{e \text{ edge of } P} l(e)l(e^{\vee}) = 24,$$

where $e^{\vee}$ denotes the edge of the polar polytope $P^{\vee}$ of P corresponding to e and l is a length-function on the edges counting the number of lattice points contained in an edge e. The proof is not easy and uses the dictionary between polytopes and toric varieties as well as results about the $K3$-surface. For a certain choice of weights, the combinatorial version of the Gauß-Bonnet-theorem can be stated for 3-dimensional polytopes P as follows:

$$\sum_{e \text{ edge of } P} \mathrm{Ric}(e) = 12 \cdot \chi(P) = 24.$$

It is tempting to claim $l(e)l(e^{\vee}) = \mathrm{Ric}(e)$ for the edges e of the reflexive lattice polytopes considered above. Interestingly enough, this does hold for easy examples but unfortunately not in general. Is there a deeper relation between these two formulae? This is a joint project with Dr. Christian Haase and was initiated during my stay at UC Berkeley.

Activities.

- Lectures and Colloquia of the graduate program

- Participation at CombinaTexas 2001, Texas A&M University, College Station, March 23-25, 2001
- Stay at Rice University (Robin Forman), Texas, March 26 - April 1, 2001
- Stay at UC Berkeley (Christian Haase), California, April 2-19, 2001
- Participation at the second Bay Area Discrete Mathematics Day, San Francisco, April 14, 2001
- Stay at the University of Minnesota (Mark de Longueville), Minnesota, April 20-28, 2001
- Participation at the graduate program's workshop at Monte Verità, May 13-15, 2001
- Talk at Rice University, title "Topics in Differential Geometry", March 27, 2001
- Talk at UC Berkeley's Combinatorics Seminar, title "Combinatorics of CW-Complexes and Differential Geometry", April 16, 2001
- Talk at University of Minnesota's Combinatorics Seminar, title "Combinatorial Curvatures of CW-Complexes", April 20, 2001
- Talk at CGC's Monte Verità workshop, title "Curvatures in Combinatorics", May 13, 2001
- Attendance at Prof. Grötschel's lecture "Theorie und Praxis der Kombinatorischen Optimierung"
- *Oberseminar Diskrete Geometrie* at TU Berlin
- Berliner Algorithmen-Tag (July 7, 2001)

Perspectives. I shall participate at DMV's Jahrestagung and at CGC's winter school on triangulations.

Part of my semester abroad will be spent between October and New Year's Eve in Zurich. I would like to thank Emo Welzl for his warm invitation.

References

[For99] R. Forman, *Combinatorial Differential Topology and Geometry* , pp. 177-207 in *New Perspectives in Algebraic Combinatorics*, L. J. Billera, A. Björner, C. Greene, R.E. Simion and R.P.Stanley, MSRI publications volume 38, Cambridge University Press, 1999.

[For00] R. Forman, *Bochner's Method for Cell Complexes and Combinatorial Ricci Curvature*, Preprint, 2000, `http://math.rice.edu/~forman/`

[Lan01] C. Lange, "Combinatorial Bochner-Laplacians, Curvatures, and Weitzenböck Formulae" , Preprint.