Semester Report Vanessa Kääb

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Field of Research: Scheduling
Topic: Local Search Algorithms
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Field of Research

In many applications such as resource constrained project scheduling the classical concept of precedence constraints fails to model the required restrictions. Therefore I am concentrating on the generalized model of AND/OR-networks. AND-nodes represent the well known standard precedence relations where OR-nodes represent so called waiting conditions. These waiting conditions model the situation that a job has to wait for the completion of one other job out of a pre-given set of alternatives. A similar concept of precedence constraints are directed hypergraphs which are studied in [GS99], for example. This generalized concept of precedence constraints has applications in resource-constrained project scheduling, but also in simple linear production systems, assembly systems, transportation models, and relational databases, for example. I am focusing on the scheduling problem with AND/OR precedence constraints so as to minimize the latest completion time of any job. Basic concepts as feasibility, transitivity and the computation of earliest start time schedules have been well investigated and efficient algorithms have been proposed for example in [MSS00], [Hva01], or [SU00].

Results

In network analysis longest paths and critical jobs are well known concepts. In AND/OR graphs their standard definition seems to be not applicable. Nevertheless it is possible to identify different kinds of critical jobs in AND/OR-networks representing the different characteristics of critical jobs in classical networks. Thus I have defined delay-, cut-, and path-critical jobs. It can be proved that cut-critical jobs are also path-critical and vice versa. Furthermore path-critical jobs are also delay-critical, but not the other way around. The property of being critical is defined through the membership in a set of critical jobs, also referred to as critical set. To find any such set in a given AND/OR-graph is easy and can be done in polynomial time. In contrast to

this the problem to identify a certifying set for a specific job is NP-hard.

In addition to this basic characteristics of critical jobs in AND/OR-networks it turned out that the system of cut-critical sets and the system of path-critical sets are clutters and that the system of cut-critical sets is the blocking clutter of the system of path-critical sets. Unfortunately they do not have the Min-Cut-Max-Flow property.

Questions like "Is the gap between the maximal flow and the minimal cut bounded?" are still unsolved and attract my attention at the moment.

Activities

- Lectures and Colloquia of the Graduate Program

- Block-Course on *Connectivity Problems of Networks: Structures and Algorithms* at FU Berlin, April 1 – 12, 2001

- 9th Workshop *Resource-Constrained Project Scheduling* in Berlin, April 2 – 4, 2001

- Workshop on *Combinatorics, Geometry, and Computation*, Monte Verità, Ascona, Switzerland, May 13 – 15, 2001

Preview

- Guest of Prof. Dr. Thomas Erlebach at the ETHZ, July 2001 – January 2002.

- Fall School on *Discrete Geometry - Triangulations from various points of view* in Berlin, October 4 – 6, 2001.

- Block-Courses on *Randomized Algorithms* and *Topological Methods in Combinatorics and Geometry* in Zurich, October 22 - November 23, 2001.

Reference

[GS99] Giorgio Gallo and Maria Grazia Scutellá. Directed Hypergraphs as a Modelling Paradigm *Technical Report* TR-99-0252, Università di Pisa, Italy, 1999.

[Hva01] Dušan Hvalica. Searching for a minimal solution subgraph in explicit AND/OR graphs. *Discrete Applied Mathematics*, 110:213–225, 2001.

[MSS00] R. H. Möhring, M. Skutella, and F. Stork.   Scheduling with AND/OR-precedence constraints. *Technical Report* 689-2000, Technische Universität Berlin, 2000.

[SU00] F. Stork and M. Uetz. On the Representation of Resource Constraints in Project Scheduling. *Technical Report* 693-2000, Technische Universität Berlin, 2000.