Semester Report Hendricus van der Holst

Supervisor:	Prof.Dr. Martin Aigner
Field of Research:	Graph Theory
Topic	Topological Graph Theory and the Colin de Verdière invariant
Postdoc Student	at the program since November 1 2000

Fields of Research

The Colin de Verdière invariant

The Colin de Verdière invariant $\mu(G)$ of a graph G, introduced by Y. Colin de Verdière in [1], is defined as the maximum corank of any matrix $M \in O_G$ with exactly one negative eigenvalue and such that it fulfills a certain transversality condition. Here O_G denotes the set of a symmetric matrices $M = (m_{i,j})$ with $m_{i,j} < 0$ if ij is an edge of G and $m_{i,j} = 0$ if $i \neq j$ and ij is a not an edge.

This invariant has the intriguing property that planar and flat graphs are described by it as those graphs G with $\mu(G) \leq 3$ and $\mu(G) \leq 4$, respectively. The open problem is to describe, if possible in topological terms, the graphs for which $\mu(G) \leq 5$. See [2] and [4].

Forbidden substructure characterization for Gauß codes on surfaces

Let C be a curve in a surfaces that lies in generic position. Label the crossing points of the curve C. The Gauß word of the curve C is the word obtained by going along C and recording the label of each crossing point when one encounters it. Two operations can be defined on a word W. If the word is of the form $W = A\alpha A\beta$, then the vertex split at A is the word $\alpha^{-1}\beta$ and the loop removal at A is the word obtained from β by deleting all labels that occur in α. If a word W comes from a curve lying in a surface S then also the vertex split and loop removal at A of W come from a curve lying in S. One can therefore ask what the minimal words are such that their corresponding curves do not lie on S. In the case that the surface is the sphere, this can been solved by Lovász and Marx [5].

Talks given

On April 19, I gave a talk at the Dutch Mathematical Conference. The title of this talk was "Embeddings and the Colin de Verdière invariant. On May 15, I gave a talk at the CGC workshop, Monte Verita. The title of this talk was knots in spatial embedded graphs. On July 13, I gave a talk at GEMS 2001, Bratislave. The title of this talk was "Embeddings and the Colin de Verdière parameter".

Papers

Revisions have been made of the papers:

- Length-Bounded Disjoint Paths in Planar Graphs,
- Graphs with Magnetic Schrödinger Operators of low Corank,
- Positive semi-definite matrices with nullity bounded by a certain number.

The first paper has been accepted for publication in Discrete Applied Mathematics.

I have been working on the papers [2, 3] and [4].

References

[1] Y. Colin de Verdière. On a new graph invariant and a criterion of planarity. In N. Robertson and P. Seymour, editors, *Graph Structure Theory*, volume 147 of *Contemporary Mathematics*, pages 137–147. American Mathematical Society, Providence, Rhode Island, 1993.

[2] H. van der Holst. Some forbidden minors for 4-dimensional flatness of graphs.

[3] H. van der Holst. An alternate proof of the characterization of graphs with Magnetic Schrödinger Operators of corank at most three.

[4] H. van der Holst, and R. Pendavingh. The Colin de Verdière parameter and 4-dimensional flatness of graphs.

[5] L. Lovasz, and M. .L. Marx. A forbidden substructure characterization of Gauss codes. *Acta Sci. Math.*, 38:115–119, 1976.